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TREND IN WEAPON TYPES AND DESIGN.*

BY

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For well over a century, The Franklin Institute has occupied a unique place in American thought. Its efforts to bring the arts and sciences to the practical use of mankind are world renowned. I feel it a special honor to be asked to address the Institute this evening on one phase of the subject of our National Defense—a subject of great concern always, and now of ever-increasing interest.

I propose to point out some of the scientific principles which, in the course of the years, have been applied to the design of weapons; and to indicate to you what we in our Army have done and are doing in our attempt to provide the Nation with the most efficient tools of defense.

When I saw Doctor Allen a few days ago, he said: "I hope you will tell us all you can." I agree to do that, within limits of time, but I am glad we have some things so new that we feel "now it cannot be told."

The story of man's effort to adapt scientific principles to self-defense is an old one. In a certain sense, it is the oldest of the arts, since civilizations have flourished only as security

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has been obtained for the family, for the tribe, for the group, and for the nation.

As the military art has progressed, leaders of armies have been constantly on the alert to see that they do not meet the enemy's weapons of today with yesterday's weapons in their own hands. They have attempted to apply the most advanced discoveries in science to their weapons of defense in order that they might be victorious in war, to allow (we hope) the nation they defend to live and to flourish in a security which in turn would permit further progress in the arts and sciences and in the pursuit of happiness.

Each of the great civilizations of history developed something new in the art of warfare. These inventions, warlike as they were, contributed to the safety of national boundaries behind which further progress in the arts of peace was possible.

Epoch-making innovations in the military art spring to mind at once:

The invention of gun powder by the Chinese.

The construction by the Romans of the Ballista and other great war machines, which depended upon the application of the principles of mechanics.

The application of gun powder to cannon and hand arms, resulting in the development of the musket which was in general use in Europe by 1550.

The application of geometry to the construction of the great medieval fortresses of Europe, when arms had become sufficiently standardized to permit the calculation of trajectories and the danger areas surrounding citadels.

Experiments with breech loading guns from 1745 to 1854.

The tremendous advances which came with the modern age of invention and the up-building of industry in the quantity production of manufactured articles.

The period since the Napoleonic wars has seen the introduction of steel and its application to gun manufacture and projectiles. The development of chemistry has led to the manufacture of propellants and explosives exceeding by many times the effectiveness of the cruder preparations of a century ago. The more recent development of the gasoline engine has also had a revolutionary effect on warfare, vastly increasing

both the tactical and strategic mobility of troops on the ground and in the air, and bringing into reality the "blitzkrieg."

It would be conservative to say that the development of military weapons during the last three generations—back only to the time of our grandfathers—has been greater than during the entire previous period of recorded history.

With each new invention has come a corresponding change in methods of warfare. A powerful weapon has added to the offense, and such a move has been countered by new devices for adding to the strength of the defense. At each stage, the soldier into whose hands the weapon was put had to devise the best tactical methods for its employment; and the form of tactical doctrines has changed accordingly. The fundamental principles of warfare are old. Armies have forever been conservative, and new weapons have always been slow of adoption. But the more aptly designed weapon, in skillful hands skillfully led, has utilized the old principles to bring victory in battle.

The dire threat of conquest has not hung over our country as it has over many other great nations. Our ocean boundaries and our considerable resources have given us good fortune in being able to develop a continent in comparative peace, and in reasonable self-sufficiency and prosperity. In spite of our relative immunity from the troubles which have beset Europe, we have too often been visited by war. On these occasions our military arm has made use of weapons designed and manufactured in the United States, although in periods of emergency, following years of inadequate preparedness, we have, in good judgment, drawn on Europe both for design and for material.

In the recent war (I refer to World War the First) when we adopted many European items of material, we were fortunate in obtaining types entirely appropriate to our needs and in quantities which we could not have built up from American production if we had confined ourselves to purely American types. Consequently, we had to choose between accepting the output of French and British factories which were already at a high level of efficient production, or of attempting to achieve in haste a quantity production at home. Fortunately, it was recognized that maximum effort could best be achieved

if we supplied raw materials to our Allies and, in turn, accepted from them finished articles which they were already producing on a large scale.

The duty of supplying weapons of war has in all armies been a most important responsibility, and the armorer has been an indispensable companion of the victor. But with the requirements of modern war, the demand for munitions has outgrown the capacities of any armorer or group of armorers. Each nation has had to utilize its entire industry for arming the military forces, and the demand for munitions has been apparently limited only by the industrial capacity of the country—the ability to pay being a very minor issue.

In our own army, the responsibility for producing the major items of munitions was in the early times assigned to the Artillery arm, since that branch appeared best able to procure and test the guns and cannon which were being provided in very limited quantities for the defense of the new United States. Over a century and a quarter ago, however, it was evident that a specialized corps, charged with the design and manufacture of weapons for the army, was a necessity; and accordingly at the beginning of the War of 1812, Congress authorized the creation of the Ordnance Department. Since that time the responsibility for equipping the army with essentially all the items of munitions which it uses in combat has rested with this service.

The mission of the Ordnance Department then may be briefly expressed as follows:

To supply our troops with Ordnance in the quantities needed and at the time and place required. More specifically, these duties embrace the design, development, manufacture or procurement, storage, issue, and maintenance of practically every offensive and defensive military weapon essential to the successful protection of our country.

In carrying out this mission, we must take cognizance of over 1200 separate items and some 250,000 components in the general categories of cannon, gun mounts, rifles, machine guns, fire control devices, tanks, bombs, pyrotechnics, fuzes, and all types of small arms and artillery ammunition. Most of these items are non-commercial in character and daily are becoming more and more complex. They must be designed by specially

trained Ordnance engineers who are not readily obtainable from industry. The Ordnance Department guards closely its reputation as a producer of weapons of great reliability.

To produce the weapons necessary for the army, we depend first on American industry and second on our great manufacturing arsenals. The Department maintains 6 of these arsenals for executing many and varied developmental projects, for producing in small quantities practically every type of non-commercial Ordnance item desired for our troops, and for developing processes and plans for their efficient manufacture. Out contact with industry is maintained both through these arsenals and through the 14 Ordnance Procurement Districts into which the country has been divided.

The Springfield Armory, Massachusetts, for example, has been in contact with the industry of its locality since 1794, when Congress passed an act which provided for the establishment of a system of government-owned shops for the manufacture of muskets; and, in fact, it can be said that the Armory has been largely instrumental in making the city of Springfield and the valley of the Connecticut River the center of the small arms industry of the country, and the Armory can lay claim to being responsible for the development of the skillful toolmakers and mechanics, and the industrial enterprises requiring their services which center in this New England community.

Here, the musket or rifle has been produced continuously for 144 years—from the original French Charlevoi Musket through the years to the present semiautomatic shoulder rifle. It was here that the old Blanchard stock-turning lathe was invented, so important to general industry. Here, too, the interchangeability of parts was developed; and here, also, was designed and manufactured the famous 1903 Springfield rifle, so universally recognized as the outstanding bolt-action military arm of the world.

Let us mention also the magnificent history of Frankford Arsenal, in your own city. Organized only twenty years later than Springfield Armory, it has carried on without interruption, in war and in peace, the knowledge and art of skillfully and loyally producing wartime material. At the moment, Frankford Arsenal is engaged in the manufacture of ball, armor piercing and tracer ammunition for rifles and machine

guns. It is turning out with high efficiency and at a high rate shell for army artillery and naval guns. It is manufacturing the mechanical time fuze, a delicate instrument of precision which determines, by a very accurate clock mechanism, the exact instant an explosive shell is to be detonated.

The expert riflemen who attend the National and International Matches at Camp Perry and elsewhere, swear by the rifle ammunition made by Frankford Arsenal and believe it to be the best obtainable. They ought to know.

In addition to fabrication, these arsenals operate research laboratories and assist greatly in the development of new designs and processes. They keep alive the art of manufacturing Ordnance and provide a nucleus of trained personnel for purposes of inspection and instruction. During the early stages of war and until industry could get into production, they would be the main source for the manufacture of Ordnance material.

The present Congress has authorized an Ordnance manufacturing program which is larger than any other program ever allowed, peace or war, except only during the World War period. Its magnitude may be realized by recalling that for the fiscal year 1938, Ordnance appropriations amounted to 25 million dollars; for the fiscal year 1939, to 50 million dollars; and for the fiscal year 1940, to 150 million dollars—a doubling and then a trebling of appropriations for re-arming the land forces. The average appropriation for the years 1922 to 1928 was about 12 million dollars; this year, twelve times as much.

You understand, of course, that our government arsenals, important as they are, are not in any position to increase their operations twelve fold; and it is not, in fact, desirable that they should do so. Our arsenals have been adequate to supply the usual needs of the Army; and, also, to act as centers of information, repositories of standards, and places where the art of munitions making can be kept alive. In time of war, they could probably produce no more than 10 per cent. of the Army's requirements; and civilian industry would be called upon to fill the remaining 90 per cent. of the demands.

Since the whole civilian industry is faced with the task of producing this huge volume of munitions in the unfortunate event of war, it is desirable that, in time of peace, it should

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become familiar, at least to some extent, with the intricacies of munitions manufacture. Earlier peace-time requirements have not been large, and there has been little incentive for industry generally to bid on munitions orders. As a result, peace-time contracts for munitions have been largely restricted to small groups of manufacturers who specialize on such items. This difficulty has been lessened by recent legislation which permits the placing of special educational-order contracts. This legislation has the objective of spreading the art of munitions making over a wider base. Under this system, small quantities of munitions items are contracted for, but the manufacturer is required to tool up on a small scale, as if for mass production, the various aids to manufacture which he finds necessary to install, the special tools, jigs and appliances, passing upon completion of the contract into the possession of the United States. The administration of the law is surrounded by proper safeguards to insure the educational features of the system.

As an example of the workings of the educational orders law, I may cite a recent contract between the government and the Winchester Repeating Arms Company of New Haven, Conn. About a year ago, this educational order was signed with the company for the production of a few units of the Army's new rifle—the Garand semiautomatic. Very recently, as a result of competitive bidding, an order was placed with the same company for 65,000 of these rifles. Because of the previous educational order, the contract price of this manufacturing order was lower by almost the exact amount to be paid to the company for the educational order; and the agreed time of performance was very much less than might have been expected. In this case, the educational order not only paid for itself in less than a year, but the time saved will become a material and very important consideration in the process of re-arming our Infantry.

Educational orders are, of course, only a means for speeding the preliminary preparation of industry for munitions manufacture. War production would be undertaken by industry on the basis of surveys made by our District Ordnance Offices; and I would like to say a word about the participation of the Philadelphia District in these plans.

Based on our peace-time planning, ordnance schedules (not orders) to the approximate value of 6 billion dollars have been placed with commercial facilities throughout the United States. Slightly over one-sixth, or 1 billion dollars, is scheduled for production in the Philadelphia District. Practically every type of ordnance can be produced within its boundaries. In this respect, this District is unique. Approximately 100 contracts for critical items of ordnance are scheduled for production here. Over 300 plants in the District have been surveyed and more than 100 are definitely allocated for war production.

The Philadelphia Ordnance District also carries a larger peace-time load than any other district, and the ability of manufacturers in this area to handle the allocated war load has been borne out by the volume of current production orders for which they have successfully bid.

Recently, the activities of the district offices have been extended to include the responsible duty of inspecting many of the current orders from industry. At the present time, this District has 80 inspection orders under its supervision. You will well understand the importance and the difficulty of establishing a qualified inspection organization to carry on the unusual load imposed by this year's orders.

The program of last year was good training for the procurement, three times as large, for the present year. This year's production will require capacity operations at all the manufacturing arsenals on a two-shift basis, and the arsenals, in addition, will act as centers of award for large contracts to civilian industry. The number of civlian employees at arsenals has already increased some 40 per cent. over last year, and it may be assumed that the contracts placed with industry have resulted in reëmployment of many thousands of skilled specialists and semi-skilled laborers.

Many items of the soldier's kit, of course, can be taken with slight or no modification directly from the products of our civilian industries. But the Army's fighting tools cannot be improvised. They are special and technical, and their design demands a highly specialized engineering talent.

Ordnance designs must not only function properly; they must be equal to or better than those possessed by any possible

adversary. They must also be susceptible of quantity production by commercial manufacturing methods.

How does the Ordnance Department go about getting its equipment so that these objectives can be met? Let me illustrate by a typical example—the light tank. How did it grow? How did it become a standard piece of equipment for our Army?

Light tank.

The tank is Infantry equipment. After many years of experience with World War tanks and succeeding types, the Chief of Infantry, some five years ago, prepared a set of what we term "military characteristics" for a light tank. These characteristics really constituted a preliminary set of performance specifications and presumably represented the maximum of performance consistent with the possibilities of production. In preparing these military characteristics, the Chief of Infantry was assisted by the Infantry Board, at Fort Benning, Georgia, and the design section of the Ordnance Office in Washington.

The military characteristics decided upon were then processed through the Ordnance Committee on their way to the

General Staff—very useful steps in the procession insuring an understanding of the combined thoughts of the using service, the technical branch, and representatives of the line of the Army. Meantime, in the Office of the Chief of Ordnance a design study was prepared.

After approval by the Staff of the military characteristics, the design study was sent to Rock Island Arsenal where manufacturing drawings were prepared, and a pilot tank pushed into manufacture.

During my few minutes recital of these events, the months have flown by.

Eventually the pilot was completed, and sent to the Aberdeen Proving Ground for a gruelling test to uncover engineering weaknesses, and to make a preliminary determination to see if the unit would fulfill the prescribed military characteristics.

The pilot tank successfully passed the Proving Ground test. It was then forwarded to the Infantry Board for an exhaustive field test in the hands of Infantry soldiers—simulating service conditions. After many difficulties had been encountered and corrected, the tank was cleared by the Infantry Board. The Chief of Infantry and the Chief of Ordnance then recommended to the Secretary of War its adoption as a "standard for limited production." Even yet we were not out of the woods. But 18 tanks were at once manufactured and subjected to weeks of further field tests by troops of the line. Having passed these tests, the tank was eventually adopted as the Army's standard, some 24 months after the final military characteristics were set up.

My history of this tank is somewhat of a fairy story, because I have not emphasized the weaknesses and failures—there were many, obscure and beset with difficulties. The pilot tank was run 16,000 miles. The entire running gear was scrapped altogether and a complete new unit manufactured. The normal development process as we follow it is slow, but it does produce good results.

A tank commander is very jealous of the good name of his unit and he properly demands the best in equipment. The combat arms are the Ordnance Department's customers; and their tastes, within reason, must be satisfied. However,

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neither the Ordnance Department nor the using arm has the final word in the adoption of a standard piece of equipment for the Army. The General Staff must pass upon the *military* aspects and The Assistant Secretary of War on the *procurement* phases of each separate item.

The tank, perhaps more than any other piece of fighting equipment, represents a joint development of industry and the Army. The tank itself has heretofore been manufactured at Rock Island Arsenal, or rather, assembled there, because approximately 55 per cent. of its cost is made up of commercial components procured directly from industry. Commercial units have been used in the tank design wherever practicable, and specially designed components only were essential to proper functioning.

Every type or model of Ordnance material goes through this process before it is finally added to our list of standard equipment.

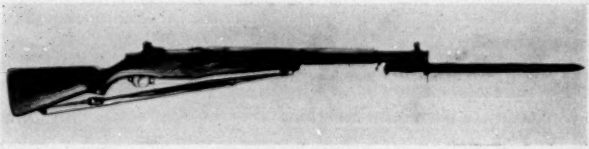
Now I should like to discuss some specific items of Ordnance that have of late been of interest to the Press, to the visitors of last month at the Aberdeen Proving Ground, and which, I hope, may be of some interest to you.

The first item that might attract our attention is the soldier's rifle—still the basic weapon of the Infantry. The shoulder arm with which many of our troops are now equipped is the Springfield Rifle, Model 1903. This rifle is internationally famous and, no doubt, known to all of you. Strangely enough, in the only war in which it was our standard arm, it had to side-step to make way for the modified Enfield rifle—a British weapon which was under manufacture in this country and which was accepted by us because it was already in quantity production in the United States and available. It was adapted to take our caliber .30 ammunition and it met our war requirements.

Today we are arming our troops with the semiautomatic shoulder rifle. This new weapon, officially known as U. S. Rifle, Caliber .30 Ml., is often referred to as the Garand rifle, because John C. Garand, an ordnance engineer employed at Springfield Armory, is principally responsible for its design and development. It is a gas operated, semiautomatic rifle

and can be fired by the average soldier approximately 2¼ times as fast as the Springfield rifle.

As long ago as 1918, Mr. Garand, a highly skilled metal worker, had a mere idea for the development of a new gun. Arrangement was made at the Bureau of Standards for Mr. Garand to work out his idea there; and his efforts made such a favorable impression on Ordnance officers who became familiar with his work that Mr. Garand was given an opportunity to accept employment at Springfield Armory, where he could further develop his design.



U. S. Rifle, caliber .30, Ml.

Garand's original rifle obtained power for operation from a rearward movement or setback of the primer, a principle, it was appreciated, that lent itself to many difficulties. In spite of this, his setback gun performed well. However, changes in powder and other conditions indicated the advisability of abandoning this principle and accepting a mechanism showing greater promise. The type of power take off which Mr. Garand then selected and has since used is based on what is known as the Bang principle, in which the power for operation is taken from the exit gases at the muzzle of the gun.

Many years were spent in experimental work and many models were constructed. The guns now in service are behaving well and are extremely popular with the soldiers to whom they have been issued. The soldier learns to use the new rifle easily, it develops a high rate of fire with less fatigue, and it is vastly superior to the old type of rifle for attack of low flying airplanes.

Of course, in designing this new weapon many problems have arisen. But the "bugs" are now out of the Garand. Quantity production is under way at Springfield, at a present rate of 100 per day, and it is expected that a production sched-

ule of 200 per day will be reached soon. We have the first military semiautomatic rifle to be adopted by any major army.

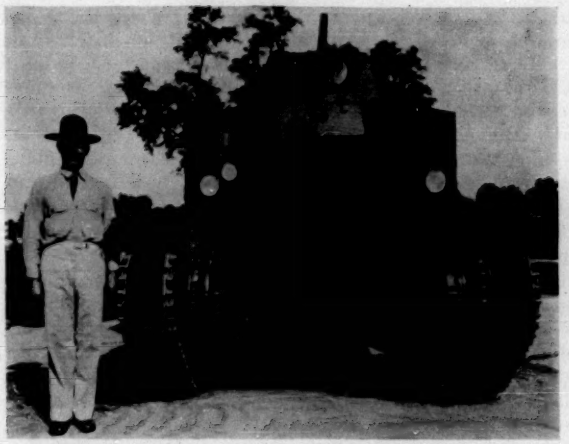
The trend in rifle design is toward the type which requires the least time of training and which gives the greatest number of aimed shots per minute with the minimum fatigue to the firer.

The trend in machine guns is toward much greater rapidity of fire and greater armor piercing qualities. The extreme rate of fire for guns mounted in airplanes is especially important, because the speed of planes makes targets so fleeting that the maximum number of shots must be fired in the briefest possible time. The trend also is toward larger calibers for special purposes.

The tank was developed during the World War for a specific purpose, and that purpose was to provide an appliance which would be capable of crossing the enemy trenches under heavy machine gun fire. It was found practically impossible in those days to break through an organized defense line and the tank was brought forth as a means of overcoming such resistance. Today the tank far transcends its original purpose. The heavy World War tank was from 22 to 30 feet long so that it could cross wide trenches, and carried about ½" of armor plate, sufficient at that time to keep out machine gun bullets. The modern tank is relatively short and capable of moving at high speed.

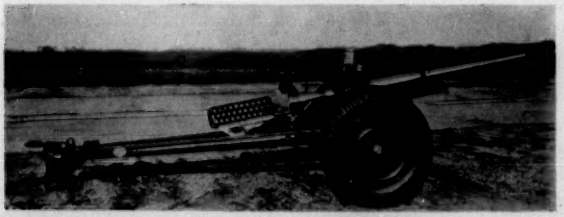
The early tank travelled at 3 or 4 miles per hour to keep pace with the marching infantry. The modern light tank dashes across country at speeds of 20 miles per hour, but is not made for spanning trenches; if the trench is wide, the tank can cross by climbing into it and out.

In addition to the light tank, all armies, including our own, employ larger and heavier tanks, also. Developmental work proceeds constantly—weight, speeds, motive power, thickness of armor, caliber .50 guns or 37 mm.—these are all problems and all complicated. They have been decided for the manufacturing program, but experimental work never stops and technical improvement is consuming much thought and time in the ordnance design sections and at the Proving Ground.



Medium tank caliber .30 machine guns in sponsons, 37 mm. gun in turret.

At the present moment, a contest is going on between the anti-tank gun and tank armor. The gun is sure to win in the long run. What use, then, is a tank whose armor can be penetrated by guns in the hands of the defending infantry? This question has brought about a change in tank tactics, resulting in the use of tanks in large groups. The present conception is that tanks are kept well back of the front lines, and thus protected, until a critical situation arises. Then they are employed boldly and in large numbers as weapons of



37 mm. Anti-tank gun and carriage.

opportunity, making a wide sweep around the enemy's weaker flank or breaking through to attack rear installations.

Quite recently, some very interesting and instructive tests were conducted at the proving ground using an electrically controlled tank as a target. Various calibers of machine guns were fired at this tank while it was moving at high speed across country. During these tests, several thousands of rounds of machine gun ammunition were fired and approximately 2,000 hits upon the tank were obtained. Up to this time, the calibers of guns and the types of ammunition were selected so that the tank would not be damaged beyond recovery. The firings demonstrated the great toughness of the tanks and an unexpected ability to continue in action after very harsh treatment. In addition to resisting penetration of bullets, we must also guard against bullet-splash. In reality, bullet-splash is volatilized lead which in its vapor form passes through doorcracks and slight openings into the vehicle, then solidifies; and is very injurious to personnel. Bullet-splash is difficult to stop. It is a problem still with us.

The trend in tank design is toward speedier tanks with heavier armament but of no greater weight. The result is always a compromise in design, a constant fight to get more done than the law of gravity allows.

The old 3-inch trench mortar of World War use did not possess sufficient accuracy to permit the shell to be reliably dropped into the enemy's trench. It was therefore less effective than required.

The 81 mm. mortar now standard in our service is greatly superior to older types. An idea of the accuracy of this weapon may be obtained from a test recently held at Aberdeen. A group of shots was fired at a range of some 500 yards. When the distance of each shot from the point of aim was measured, it was found that the maximum deviation of the group of 5 shots was only 4 yards. This shows the ability of one weapon to place the projectile where you want it.

The Infantry Board at Fort Benning recently conducted a very interesting comparative test between the current 81 mm. mortar and the caliber .30 machine gun. Firings were conducted to determine the relative efficacy of the two weapons at ranges beyond 1,500 yards upon concealed targets, such as a

target located behind a slope. The proportion to give equivalent weights in ammunition is 1,000 rounds of caliber .30 to 8 rounds of 81 mm. The effect of the fire was measured by counting the holes in silhouette targets. Under these particular conditions of long range fire, it was determined that fire from the mortar was almost three times as effective as from the machine gun. This does not of course mean that the

81 mm. Mortar in action.

machine gun would be superseded by the mortar, each having its separate function. The machine gun is still the king of all weapons for resisting attack at short ranges.

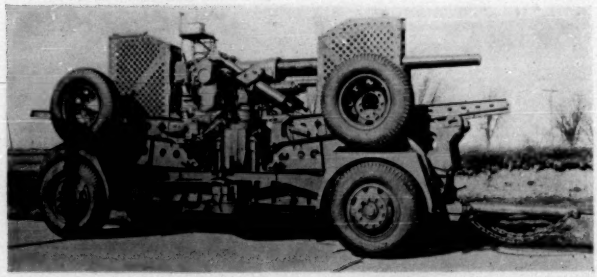
In addition to the 81 mm. mortar, the lighter and smaller 60 mm. mortar is adopted for infantry use.

The trend in this matter is toward more liberal issue of mortars to infantry and cavalry units.

The trend in antiaircraft armament is toward larger cali-

bers and higher ceiling, with minimum diminution in rate of fire; and faster and better means of fire control and direction.

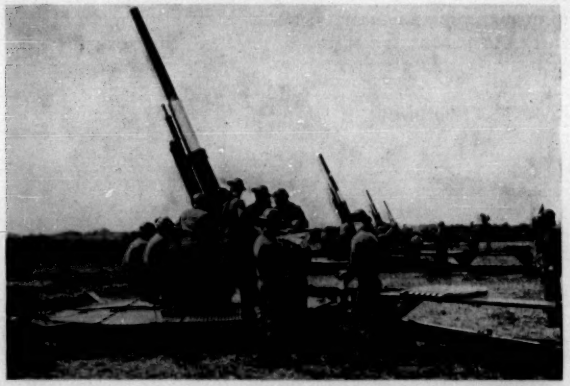
The fire control equipment for antiaircraft batteries is of special interest since antiaircraft fire, more than any other, requires the use of the most scientific methods. It is very difficult to hit an airplane target moving through space in three dimensions and at great velocity. Success can only be achieved with antiaircraft guns against such a target if exact and accurate methods of solving the fire control problem involved are utilized.



3 in. Antiaircraft gun in traveling position.

The fire control system for directing the fire of the gun at the target is the G. H. Q. of the antiaircraft battery. Prior to 1926, guns were aimed by means of sights placed on the gun carriages. The data for setting the sights were manually computed through several separate instruments. After the answers had been obtained by compilation of the results secured from these various instruments, these data were telephoned to the gunners, and the sights then set. The guns were either laid to shoot directly at the target, or an attempt was made to establish a curtain of flying fragments through which it was anticipated the plane would have to move. By continuously firing the gun and keeping the barrage ahead of the plane, it was expected that destructive hits would be made. Calculations of the probability of hitting proved the fallacy of this idea. It was found that hundreds of thousands of rounds of ammunition would be required theoretically to

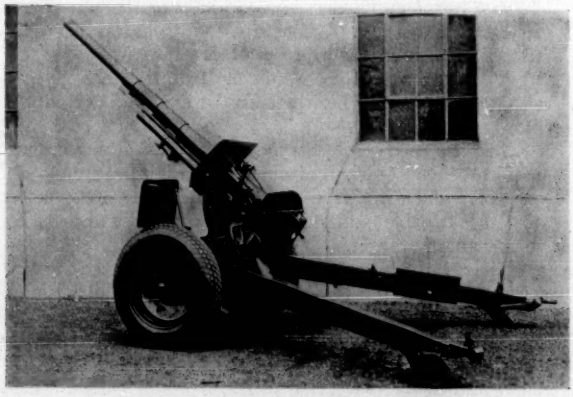
accomplish the sought-for results. This so-called "cloud method" of fire control was thus abandoned and the present method, based upon using the gun as an instrument of precision, was substituted.



3 in. Antiaircraft guns in action.

The requirement for a better system brought about the development of the director, a mechanical computer designed to calculate continuously the firing data required by the guns of an antiaircraft battery. This director is perhaps the most complicated mechanical instrument manufactured for any military purpose. It appears to be a box, mounting two telescopes for use by the gunners in tracking the target. The act of operating these telescopes continuously sets up, within the instrument, rates of travel of the airplane in the vertical and horizontal planes, thus generating its path in space and determining its instantaneous position and ground speed. The director adds the ballistic corrections due to the action of wind, drift, and muzzle velocity, and generates continuously the azimuth, elevation, and fuze range corresponding to the set forward position of the target. These data are transmitted to the guns by an electrical transmission system. This is a new kind of fire control, exceeding in speed and accuracy anything heretofore developed for the artillery.

In Divisional and Corps artillery; namely, 75 mm. guns, 105 mm. howitzers, 155 mm. guns and howitzers, and 8-inch howitzers, the trend is toward greater mobility and greater range and power. The development of the split trail has permitted high angle fire for the 75's. A vast improvement in methods of manufacture has resulted from the centrifugal casting of guns and from the extensive use of welding in carriage construction.



75 mm. gun and carriage.

Ammunition is really a complicated subject. It involves problems relating to delay, non-delay, quick and super-quick fuzes, shrapnel, high explosive shell, smoke shell, gas shell, illuminating shell, bombs for demolition, bombs for personnel effect, bombs for penetration, and bombs for superquick action. Every requirement introduces an involved problem, the designer having to bear constantly in mind that simplicity is one of the greatest of military virtues.

The trend in ammunition design is toward effecting a complete shell design with a fuze system of the minimum number of types; toward using the same contour of shell and fuzehead so that one firing table only will be required for each gun; toward improving powders—smokeless, flashless, non-

hygroscopic—to increase range, and to obtain plunging fire with superquick fuze wherever required.

As in the rifle, the tank, and the shell, many scientific problems are also involved in the design of airplane bombs.

In conclusion, may I point out the general characteristics of modern warfare which were unknown or not realized before the last world war:

The relatively great increase in the strength of the defense has demanded the provision of artillery and artillery ammunition for attack in quantities far exceeding those known in the warfare of other centuries.

The added mobility which motorization has given to all armies requires high speeds and great reliability in motor vehicles.

The increasing role played by aircraft in modern war requires the provision of efficient antiaircraft guns and controls; and aircraft armament, including weapons on airplanes capable of ultrarapid fire, and bombs appropriate to any enemy target.

The relatively greater difficulties of the attack against well prepared works make more necessary the provision of armored tanks and combat cars, by means of which infantry and cavalry fire power is advanced to and beyond hostile lines. The great progress recently made in metallurgy and in the perfection of gasoline and Diesel engines permits the production of well-armored, highly mobile tanks carrying weapons which can put in peril enemy strong points and even enemy artillery.

The needs of the individual rifleman remain paramount, and in his hands must be placed the most efficient possible weapon for direct attack on the individual rifleman of the enemy. This is accomplished by providing a magazine rifle of great power and with a rate of fire far exceeding that provided in any previous stage of warfare.

In fine, the present trends in Ordnance design are to continue to develop and to supply increasingly efficient defensive and offensive tools and mechanisms, as may be required to accomplish the Army's mission.

CHARACTERISTICS OF THE ROCKET MOTOR UNIT BASED ON THE THEORY OF PERFECT GASES.

BY

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INTRODUCTION.

Recently a descriptive survey of reaction propulsion was presented by A. Ananoff [1] which gives a clear perspective of the progress that has been made during the last twenty years. During this period a number of investigators, among them Goddard,[2] Esnault-Pelterie,[3] Oberth,[4] Sänger,[5] Rinin,[6] Buckingham,[7] Africano,[8] Vogelpohl,[9] and others have studied at length the general theory of operation of the constant pressure rocket motor unit. Most of these authors have confined their analyses within the bounds of the theory of perfect gases; there remain a number of important characteristics of this propulsive unit that can be usefully discussed within these bounds before a study, based on actual operating conditions, is made.

It is found that if the formulae expressing the operation of the ideal rocket motor are transformed into dimensionless form, a universal thrust diagram can be constructed corresponding to the fixed value chosen for the ratio of the specific heats of the products of combustion. From the diagram the thrust characteristics of the motor unit can be determined for any divergent type nozzle when the rocket is operated at an arbitrary altitude.

It must be realized that the results obtained on the basis of perfect gas theory will be subject to some alteration when the roles of such phenomena as vaporization of the propellants within the combustion chamber, variation of specific heats with temperature, dissociation of the products of combustion, heat losses, friction, and turbulence are taken into account.

In this study the general formulae that appear will be evaluated for the combination of liquid oxygen and gasoline.

This combination of propellants has been accepted by most experimentors as the one that is most favorable when all factors affecting the choice of propellants are considered. They have been chosen by Goddard, who has made a large number of experiments with them, both in static test stands and in flight.

GENERAL.

The term, rocket motor, has been attached to a device which liberates the available heat energy of combustion of a fuel and converts it into kinetic energy of flow in the products of combustion. Two types of rocket motors have been developed: (*a*) The constant pressure rocket motor uses propellants such as gases, liquids, or slow-burning dry fuels, which are burned in a combustion chamber at constant or nearly constant pressure. The thrust force is delivered as long as propellants are supplied to the motor. (*b*) The constant volume rocket motor uses as a propellant a fast-burning powder in the form of single charges fed to the combustion chamber. The duration of combustion is so short that the burning process can be assumed to take place at constant volume and the thrust force is an impulsive force (See Ref. 10). In this paper no further mention will be made of the constant volume rocket motor.

To obtain reaction propulsion from the two propellants, liquid oxygen and gasoline, an apparatus consisting of propellant tanks, feed system, mixture control, combustion chamber, and exhaust nozzle are required. The constant pressure rocket motor unit is made up of the combustion chamber and exhaust nozzle. The thermal efficiency of reaction propulsion must take into account each component of the complete system. In the past it has been common to regard the thermal efficiency of the motor unit alone as the efficiency of the propulsive system, which helps to account for the exceedingly favorable comparisons obtained with respect to other thermal engines. The assumption that the motor efficiency is the principal propulsive parameter is justified if the propellants are fed to the combustion chamber by pressure cells that are charged before flight is begun. However, the work required to charge the cells should not be neglected if comparison is made with other types of heat

engines. If feed pumps must be used during flight, the work necessary to operate them cannot be overlooked in calculating the overall thermal efficiency of the system.

It is important to stress the significance of the thermal efficiency of the motor unit in rocket propulsion. If the reaction system is to be used to propel a sounding rocket, the overall thermal efficiency of a system using pressure-cell feed can be replaced by the thermal efficiency of the motor unit, especially if the sounding rocket is to reach as high an altitude as possible. In this case the performance of the rocket will not depend on the efficiency of the work done on the ground, but only on the efficient transformation of the heat energy into as high an effective exhaust velocity as possible.

THE PROPELLANTS.

The constant pressure rocket motor system appears to possess advantages over other forms of propulsion in being able: first, to deliver a large thrust, which can be varied with great flexibility; second, to operate *in vacuo* as long as an oxidizer is carried to burn the fuel.

The first characteristic has caused the suggestion to be made that the rocket motor be used to improve certain phases of heavier-than-air performance, such as take-off and climb. To deliver a large thrust large quantities of combustibles must be consumed—a requirement that practically eliminates the use of the gaseous form of combustibles, as the supply tanks would be excessively large and heavy. If the second characteristic of the rocket motor is to be exploited in the sounding rocket, performance studies show that it is necessary that supply tanks be as small and as light as possible. For these reasons liquid propellants are the ones most likely to be useful.

The fuel used should have a high heat content and require a minimum of oxidizer to burn it completely. In other words, the heat content per pound of fuel and oxidizer necessary for its combustion should be high.

A liquid fuel can be chosen from a number of cheap, easily handled, and readily obtainable hydrocarbons. Gasoline appears to be the most general choice. To burn the fuel liquid oxygen is the most desirable oxidizer.

The characteristics of gasoline vary considerably depend-

ing on the nature of the crude oil and the process of preparation. Gasoline consists principally of a mixture of three different series of hydrocarbons:

(*a*) Paraffins, C_nH_{2n+2},
(*b*) Naphthenes, C_nH_{2n},
(*c*) Aromatics, C_nH_{2n-6}.

For thermodynamic analysis it has been found that gasoline can be closely represented by octane, C_8H_{18}. Therefor this chemical formula will henceforth be used. A value of 18,800 B.t.u./lb. has been chosen for the lower heat of combustion of the fuel.

THEORY OF THE ROCKET MOTOR WITH PERFECT GASES.

Throughout this analysis it is assumed that the perfect gas relations apply to the processes encountered in the rocket motor unit, that is, the specific heats of the products of combustion once chosen are considered constant. Furthermore, it is assumed that none of the heat liberated in the chamber is lost through dissociation or by conduction and radiation through the walls of the chamber and of the nozzle or through friction, turbulence, and condensation of the gases in the nozzle. The heat of vaporization is also neglected.

On the basis of these assumptions the chemical equation for the combustion process can be written:

$$C_8H_{18} + 12.5O_2 \rightarrow 8CO_2 + 9H_2O. \qquad (1)$$

To burn one pound of gasoline 3.51 pounds of oxygen are required. The products of combustion consist of 3.09 pounds of carbon dioxide and 1.42 pounds of water vapor. During this reaction the heat of combustion of the fuel is liberated and appears in the form of increased temperature in the products of combustion.

The specific heats of the products of combustion vary greatly with the temperature. A study of their variation with temperature shows that their values increase with the temperature. Since the gas is at high temperature during most of its stay in the rocket motor, a fairly low value of the ratio of $c_p/c_v = \gamma$ can be expected. For the propellants oxygen and gasoline a value of 1.2 has been chosen for γ.

Furthermore it is assumed that the values of c_p and c_v in B.t.u./mol./° F. are the same for both CO_2 and H_2O. Therefore for the products of combustion $8CO_2$ and $9H_2O$ with

$$\gamma = 1.2$$

and

$$R = 1.986 \text{ B.t.u./mol./° F.} = 0.0658 \text{ B.t.u./lb./° F},$$

we obtain

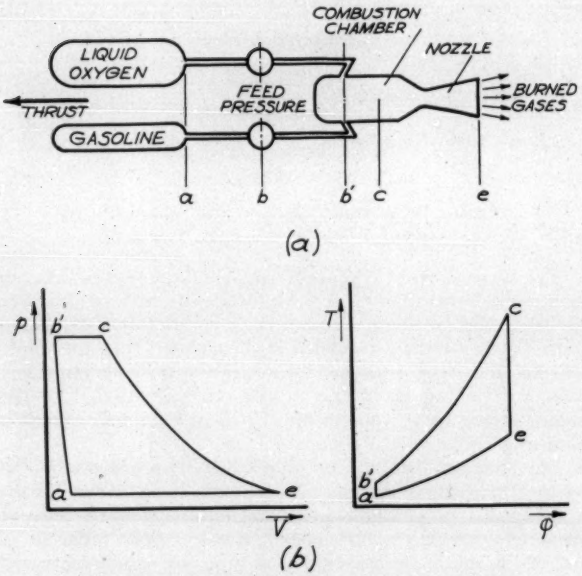
$$c_p = 11.916 \text{ B.t.u./mol./° F.} = 0.395 \text{ B.t.u./lb./° F.},$$
$$c_v = 9.930 \text{ B.t.u./mol./° F.} = 0.329 \text{ B.t.u./lb./° F.}$$

The second law of thermodynamics states that of a given quantity of heat applied to a working substance only a part can be transformed into useful energy. Before attempting to use the heat energy available in the gasoline it is, therefore, desirable first to determine the maximum possible efficiency obtainable in the ideal case of the engine. An aid to the understanding of the operation of a heat engine is found in discussing its cycle of operations.

The diagram in Fig. 1*a* shows the components of the reaction propulsion system. Consider a mass of propellant m, composed of gasoline and sufficient oxygen to burn it completely, flowing per second from the supply tanks at a. Between a and b' the pressure of the propellants is increased and vaporization is completed. At b' the propellants are fed into the combustion chamber in the state $p_{b'}$, $V_{b'}$, and $T_{b'}$. From b' to c the mixture is burned at constant pressure and the available heat of the gasoline is liberated. The state of the products of combustion, CO_2 and H_2O, is determined by p_c, V_c, and T_c. At c the burned gases enter the exhaust nozzle and the liberated heat is transformed into kinetic energy of flow by an isentropic expansion process. The jet energy acquired will be equal to the isentropic drop in heat content of the products of combustion. The state of the gases at e is determined by p_e, V_e, and T_e. The remainder of the cycle can be imagined to be completed by nature.

The cycle is made up of two isentropic processes and two constant pressure processes (Fig. 1*b*). The work done during the cycle is represented by the area enclosed by $ab'ce$ in the

FIG. 1.



Components of the reaction propulsion system and the thermodynamic cycle.

temperature-entropy diagram. The available heat liberated during the combustion process is expressed by:

$$H_p = mgc_p(T_e - T_{b'}) = mgc_p\Delta T, \qquad (2)$$

where H_p = lower heat of combustion of a unit weight of propellant mixture, B.t.u. per lb. at constant pressure,

c_p = specific heat constant of the products of combustion at constant pressure, B.t.u. per lb. per degree Fahrenheit,

T = temperature, ° F. abs.

For the propellants considered

$$\Delta T = 10{,}580° \text{ F.} \qquad (3)$$

Heat is rejected from e to a at constant pressure so that

$$H_e = mgc_p(T_e - T_a). \tag{4}$$

The ideal cycle efficiency is given by the relation

$$\eta_i = \frac{\text{Work done in B.t.u.}}{\text{Heat available in B.t.u.}} = \frac{H_p - H_e}{H_p}$$

$$= 1 - \frac{T_e - T_a}{T_c - T_{b'}}. \tag{5}$$

If it is assumed that $T_a \ll T_e$ and $T_{b'} \ll T_c$ so that T_a and $T_{b'}$ can be neglected, then

$$\eta_i = \frac{T_c - T_e}{T_c}, \tag{6}$$

but the ideal gas law $pV = RT$ gives

$$T_c = \frac{p_c V_c}{R} \quad \text{and} \quad T_e = \frac{p_e V_e}{R}. \tag{7}$$

Therefore,

$$\eta_i = \frac{p_c V_c - p_e V_e}{p_c V_c}. \tag{8}$$

Since the expansion from c to e is isentropic, the following relation holds:

$$p_c V_c^{\gamma} = p_e V_e^{\gamma} \tag{9}$$

and finally

$$\eta_i = 1 - \left(\frac{p_e}{p_c}\right)^{(\gamma-1)/\gamma}. \tag{10}$$

For the products of combustion of gasoline and oxygen with

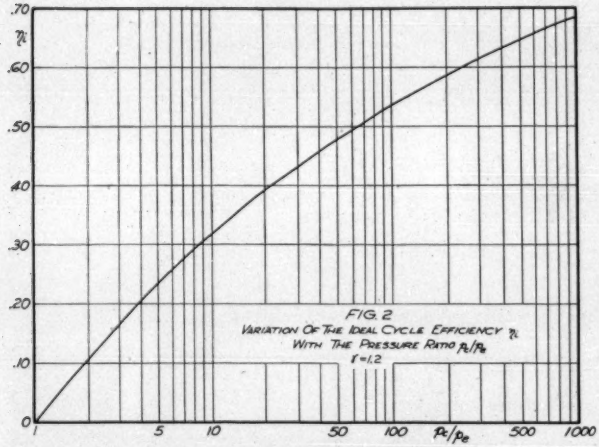
$$\gamma = 1.2 \tag{11}$$

it follows that

$$\eta_i = 1 - \left(\frac{p_e}{p_c}\right)^{0.167}. \tag{12}$$

This relation for the ideal cycle efficiency shows that the efficiency increases with the expansion ratio, i.e., with an increasing chamber pressure and a decreasing back pressure. In Fig. 2 the variation of the ideal cycle efficiency with the pressure ratio is shown.

FIG. 2.



Variation of the ideal cycle efficiency η_i with the pressure ratio p_c/p_e.

The experimenter has found it more convenient to define a thermal efficiency η_{th} of the rocket motor unit in terms of the kinetic energy of the outflowing products of combustion and the available heat energy of the fuel; thus the familiar form is:

$$\eta_{th} = \frac{A(w_f + w_{o_2})c^2}{2gw_f H_{p_f}}, \qquad (13)$$

where A = mechanical equivalent of heat $= \frac{1}{777.6}$ B.t.u./ft. lb.,

w_f = weight of fuel in lbs. burned per sec.,

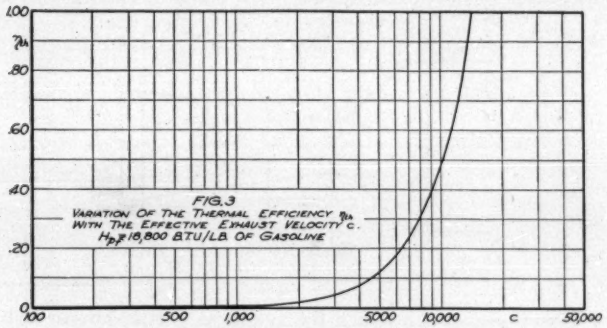
w_{o_2} = weight of oxygen in lbs. used per sec.,

c = effective exhaust velocity, ft. per sec. (the quantity c will be discussed later),

H_{p_f} = lower heat of combustion of the fuel, B.t.u. per lb. at constant pressure.

In Fig. 3 the thermal efficiency η_{th} is plotted against the effective exhaust velocity c for the ideal oxygen-gasoline motor.

FIG. 3.



Variation of the thermal efficiency η_{th} with the effective exhaust velocity c.

IDEAL TRANSFORMATION OF AVAILABLE HEAT ENERGY.

The first correct treatment of the elementary theory of the discharge of gases under high pressure was published by Saint-Venant and Wantzel. The theory has since been extended and applied in the de Laval nozzle. The de Laval divergent nozzle converts the available heat energy liberated in the combustion chamber into kinetic energy of flow of the exhaust gases. (See Ref. 11.)

This conversion of energy is most complete when an isentropic expansion takes place in the nozzle. The theory of the de Laval nozzle is so well known that the formulae employed will not be derived. However, the formulae will be expressed in dimensionless form which will make possible the construction of a universal thrust diagram for the ideal rocket motor unit using any chosen propellants. The diagram is universal in that from the diagram constructed for the fixed value of γ the propulsive characteristics of the motor unit with any de Laval type nozzle can be immediately determined.

In Fig. 4 a diagram of a divergent type nozzle is shown with the notation used for its dimensions. The rate of flow of the products of combustion under a pressure p_c in the combustion chamber is controlled by the existence of a critical pressure p_t at the nozzle throat. The throat is the narrowest

FIG. 4.

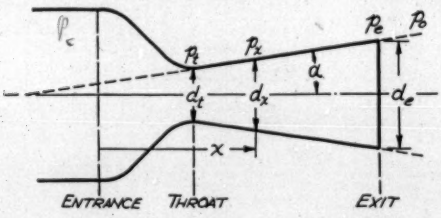


Diagram of a divergent type nozzle showing notation used for its dimensions.

cross-section through which the gas flows. It is known that the rate of flow of a gas through the throat will increase up to the point that the exit pressure reaches the critical pressure at the throat, thereafter the rate of flow will remain constant. It will be assumed that the critical pressure always prevails in the throat. The ratio of the critical pressure to the chamber pressure is given by

$$\frac{p_t}{p_c} = \left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma-1)}. \tag{14}$$

If the critical pressure prevails at the throat, the velocity of flow of the gas through it will be equal to the sonic velocity of the gas corresponding to its state in the throat. The velocity of flow through the throat can also be expressed in terms of the conditions in the combustion chamber:

$$v_t = \sqrt{\frac{2gR\gamma T_c}{\gamma + 1}}. \tag{15}$$

The velocity of sound corresponding to the conditions in combustion chamber is expressed by the relation

$$a_c = \sqrt{gR\gamma T_c}, \tag{16}$$

so that eq. (15) can be written in dimensionless form

$$\frac{v_t}{a_c} = \sqrt{\frac{2}{\gamma + 1}}. \tag{17}$$

The rate of flow of propellants is given by

$$w = \frac{f_t v_t}{V_t} = \left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/2(\gamma-1)} \frac{f_t p_c a_c}{RT_c}, \tag{18}$$

where f_t = area of cross-section of the nozzle throat.

As the flow in the nozzle has been assumed to follow an isentropic process, the following expression can be written for the conservation of energy:

$$c_p T_c = \frac{v_x^2}{2g} + c_p T_x, \tag{19}$$

so that the velocity at any cross-section is given by

$$v_x = \sqrt{2gc_p T_c \left(1 - \frac{T_x}{T_c}\right)} \tag{20}$$

or

$$\frac{v_x}{a_c} = \sqrt{\frac{2}{\gamma - 1}\left(1 - \frac{T_x}{T_c}\right)}; \tag{21}$$

also

$$\frac{v_x}{a_c} = \sqrt{\frac{2}{\gamma - 1}\left[1 - \left(\frac{p_x}{p_c}\right)^{(\gamma-1)/\gamma}\right]}. \tag{22}$$

This equation shows that the liberated heat energy will be completely transformed into kinetic energy of flow when the pressure at the exhaust section is zero. Thus the ideal transformation of the heat energy is the optimum when the rocket motor is operated *in vacuo*.

Heretofore it has been assumed that the gas flowing through the exhaust nozzle has either been completely expanded to the external pressure or that expansion has not been completed so that the gas passes the exit cross-section at a pressure higher than the external pressure. If, however, the expansion ratio f_x/f_t is too great, overexpansion of the gas will result and if the overexpansion is continued a shock wave may occur.

It is possible that before overexpansion takes place the jet will separate from the walls of the nozzle, in which case the most efficient transformation of the available heat energy into velocity will not be affected. It will be assumed that the jet does not separate; hence overexpansion to some pressure below the external pressure is possible.

Experiments with gases flowing at high velocity in an expanding nozzle show that if overexpansion is continued, when the external pressure p_0 is greater than zero, at some pressure p_{s_2} a sudden change in the characteristics of the gas flow will result. The flow characteristics again reach a steady state after passing through a narrow transition layer. The thickness of the transition layer is extremely small so that it can be assumed that there is a discontinuity in the flow characteristics. The transition layer is known as a "shock wave." (For the basis of this discussion and of the following analysis see Ref. 12.)

If the divergence angle of the nozzle is small, it can be assumed that the shock wave is a "plane shock wave," i.e., the change in flow characteristics takes place across a plane perpendicular to the nozzle axis.

Denoting for all points on the inner side of the wave the velocity, pressure, and mass density by v_{s_1}, p_{s_1}, and ρ_{s_1}, $\left(\rho = \frac{1}{Vg}\right)$, and on the outer side by v_{s_2}, p_{s_2}, and ρ_{s_2}, the equation of continuity can be written:

$$\rho_1 v_{s_1} = \rho_2 v_{s_2} = m_a, \tag{23}$$

where m_a is the mass that crosses a unit area of the shock wave per second. The equation for the conservation of momentum is

$$p_{s_1} - p_{s_2} = m_a(v_{s_1} - v_{s_2}), \tag{24}$$

and the condition of the conservation of energy is

$$p_{s_1}v_{s_1} - p_{s_2}v_{s_2} - \frac{1}{2}m_a(v_{s_2}^2 - v_{s_1}^2) = \frac{m_a}{\gamma - 1}\left(\frac{p_{s_2}}{\rho_{s_2}} - \frac{p_{s_1}}{\rho_{s_1}}\right). \tag{25}$$

Making use of eq. (23) and eq. (24) the energy equation

can be written in the form:

$$\frac{1}{2}(p_{s_1} + p_{s_2})\left(\frac{1}{\rho_{s_1}} - \frac{1}{\rho_{s_2}}\right) = \frac{1}{\gamma - 1}\left(\frac{p_{s_2}}{\rho_{s_1}} - \frac{p_{s_1}}{\rho_{s_2}}\right). \quad (26)$$

This relation is known as the Rankine-Hugoniot equation. Solving for ρ_{s_1}/ρ_{s_2} the equation is obtained in the form:

$$\frac{\rho_{s_1}}{\rho_{s_2}} = \frac{(\gamma - 1)p_{s_1} + (\gamma + 1)p_{s_2}}{(\gamma - 1)p_{s_2} + (\gamma + 1)p_{s_1}}. \quad (27)$$

From eq. (23):

$$\frac{v_{s_1}}{v_{s_2}} = \frac{\rho_{s_2}}{\rho_{s_1}} \quad (28)$$

and this relation together with eq. (24) gives upon substituting into eq. (27)

$$\frac{p_{s_2}}{p_{s_1}} = \frac{2\gamma}{\gamma + 1}\left(\frac{v_{s_1}}{a_{s_1}}\right)^2 - \frac{\gamma - 1}{\gamma + 1}, \quad (29)$$

where

$$a_{s_1} = \sqrt{\frac{p_{s_1}\gamma}{\rho_{s_1}}} = \text{velocity of sound corresponding to the conditions of the gas on the inner side of the shock wave.} \quad (30)$$

Since an isentropic process has been assumed up to the point that the shock wave occurs, equation (26) can be expressed in terms of the chamber pressure:

$$\frac{p_{s_2}}{p_{s_1}} = \frac{4\gamma}{\gamma^2 - 1}\left[\left(\frac{p_c}{p_{s_1}}\right)^{(\gamma-1)/\gamma} - 1\right] - \frac{\gamma - 1}{\gamma + 1}. \quad (31)$$

By means of eq. (31) the pressure p_{s_1} to which the gas must be expanded to cause a shock wave with the pressure p_{s_2} on the outer side can be determined. Further, the pressure p_{s_2} cannot exceed the external pressure. For the rocket motor unit p_{s_2} cannot be greater than the pressure p_0 that prevails at the altitude at which the rocket is being operated. The value of p_{s_1} will be the greatest when $p_{s_2} = p_0$, the atmospheric pressure at sea level. Therefore at sea level the shock wave will occur the closest to the nozzle throat. The cross-section at which the shock wave first occurs will be called the "critical shock cross-section."

To determine the area of the "critical shock cross-section" $f_{x_s}^*$, the pressure p_{s_1} is calculated for $p_{s_1} = p_0$ (this is done most easily graphically). Then $f_{x_s}^*$ is obtained from the relation

$$\frac{f_x}{f_t} = \frac{\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}}{\left(\frac{p_x}{p_c}\right)^{1/\gamma}\sqrt{\frac{2}{\gamma-1}\left[1-\left(\frac{p_x}{p_c}\right)^{(\gamma-1)/\gamma}\right]}} \tag{32}$$

by putting $f_x = f_{x_s}^*$ and $p_x = p_{s_1}$. Equation (32) can be derived from the equation of state

$$f_x = \frac{wRT_x}{v_x p_x} \tag{33}$$

and the relations for an isentropic expansion.

If $f_{x_s}^*$ is greater than the area of the exit section f_e, of the nozzle, the shock wave will occur outside of the nozzle and the characteristics of the flow in the nozzle are those corresponding to an isentropic expansion from the combustion chamber. If $f_{x_s}^* = f_e$ the characteristics of the flow will be affected at the exit cross-section. If $f_{x_s}^* < f_e$ the gas will expand further, disregarding the possibility of separation, and give rise to a shock wave for which $p_{s_1} < p_0$ so that the gas will be compressed after the shock wave has been passed. The compression of the gas cannot, however, raise the pressure at the exit cross-section above the external pressure. With compression after the shock wave the shock wave will occur closest to the throat when the gas is just compressed to the external pressure at the exit cross-section. In Fig. 5 a diagram shows the various regions discussed. (See Ref. 11.)

The characteristics of the flow in the region following a shock wave can be determined by making use of the relations for isentropic compression. The velocity of flow at the exit section if $f_e > f_{x_s}^*$ is simply determined using the equation of state:

$$v_{x_s} = v_e = \frac{wRT_e}{f_e p_0}, \tag{34}$$

and the equation for the conservation of energy,

$$c_p T_c = c_p T_e + \frac{1}{2g} v_e^2, \tag{35}$$

FIG. 5.

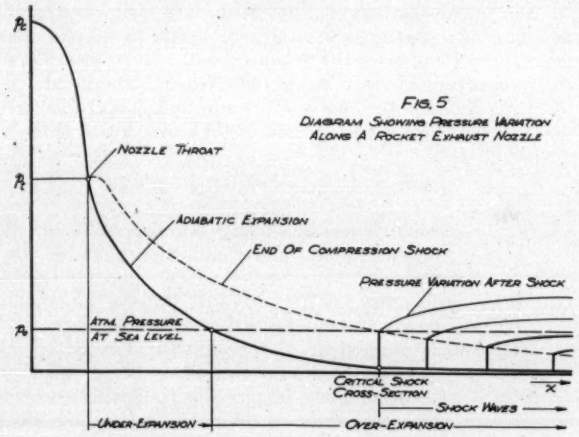


Diagram showing pressure variation along a rocket exhaust nozzle.

which give upon evaluating T_e in eq. (34) from eq. (35), substituting eq. (18) for w, and remembering the relation for a_c:

$$v_e^2 + \frac{2a_c f_e p_0}{(\gamma - 1)\left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/2(\gamma-1)} f_t p_c} v_e - \frac{2a_c^2}{\gamma - 1} = 0. \tag{36}$$

Solving for v_e and writing it in dimensionless form:

$$\frac{v_e}{a_c} = -\frac{\left(\frac{\gamma + 1}{2}\right)^{(\gamma+1)/2(\gamma-1)}}{\gamma - 1} \frac{f_e p_0}{f_t p_c} + \left[\frac{\left(\frac{\gamma + 1}{2}\right)^{(\gamma+1)/(\gamma-1)}}{(\gamma - 1)^2}\left(\frac{f_e p_0}{f_t p_c}\right)^2 + \frac{2}{\gamma - 1}\right]^{1/2}. \tag{37}$$

The solution of the quadratic relation (36) gives for the second term a $\pm$ sign. If the minus sign is chosen, the velocity v_e will be supersonic, which is not possible in the flow with compression after the shock wave. Therefore the positive sign has been retained, which assures a subsonic flow.

The transformation of heat energy into kinetic energy of flow and into pressure causes a force to act in the direction opposite to the flow of the exhaust gases. By application of the momentum theorem to the flow through the nozzle the following relation between the pressure forces and the force due to the change in momentum of the gases can be written:

$$\int p dS - \int (p_e - p_0) d f_e = \int v_{e_x} dm, \tag{38}$$

where p = absolute pressure acting on the walls of the exhaust nozzle and combustion chamber, lbs. per sq. in.

dS = elementary surface area of the walls of the exhaust nozzle and combustion chamber, sq. in.

v_{e_x} = axial-component of the exhaust velocity at the exit cross-section of the nozzle, ft. per sec.

dm = element of mass of propellants flowing through the exit section of the nozzle per second, slugs per sec.

The first integral is to be taken over the whole surface area of the exhaust nozzle and combustion chamber. The second and third integrals are taken over the exit section of the nozzle only.

The first integral corresponds to the thrust of the rocket motor, which is designated by $F = \int p dS$, and the third integral is given by $(p_e - p_0) f_e$, so that

$$F = \int_{f_e} v_{e_x} dm + (p_e - p_0) f_e. \tag{39}$$

If it is assumed that the flow at the exit section is uniformly radial * (see Fig. 6), then

$$v_{e_x} = v_e \cos \theta. \tag{40}$$

* The author is indebted to Dr. H. S. Tsien for the analysis leading to an expression for λ.

The mass of propellants flowing through a ring of width $Rd\theta$ on the spherical surface f_e' is expressed by

$$dm = \rho v_e df_e'. \tag{41}$$

FIG. 6.

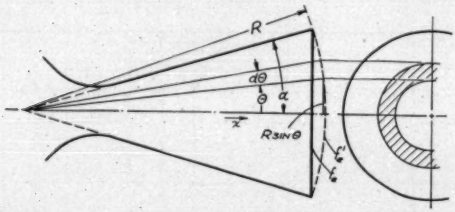


Diagram showing notation used for various quantities used in nozzle flow analysis.

Therefore,

$$\int_{f_e} v_{e_x} dm = 2\pi R^2 \rho v_e^2 \int_0^\alpha \sin\theta \cos\theta d\theta; \tag{42}$$

integrating and expressing R in terms of f_e',

$$\int_{f_e} v_{e_x} dm = \rho v_e f_e' \cdot v_e \cdot \frac{1 - \cos 2\alpha}{4(1 - \cos\alpha)}. \tag{43}$$

However, $\rho v_e f_e' = m$, the mass of propellant flowing through the exit section per second, and if we write $\lambda = \dfrac{1 - \cos 2\alpha}{4(1 - \cos\alpha)} = \dfrac{1}{2} + \dfrac{1}{2}\cos\alpha$, then

$$\int_{f_e} v_{e_x} dm = \lambda m v_e. \tag{44}$$

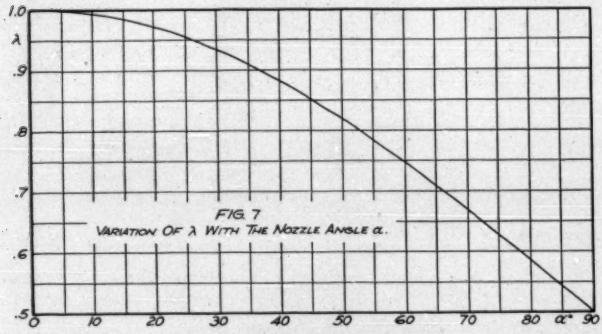
Hence, the thrust of the rocket motor can be written in the form:

$$F = \lambda m v_e + (p_e - p_0) f_e. \tag{45}$$

It is seen that only the momentum term is affected by the angle of divergence of the nozzle. In Fig. 7 the variation of λ with the nozzle angle α is shown. The small decrease in λ up to divergent angles of 35° is worthy of notice as it may permit an efficient use of shorter nozzles for a correct expansion ratio. This makes possible a reduction in the weight of

the nozzle and in the surface area of the nozzle through which heat losses occur. Therefore an optimum nozzle for the rocket motor will probably have a much larger divergent angle than is usual in the design of the de Laval steam nozzle.

FIG. 7.



Variation of λ with the nozzle angle α.

Equation (30) for the thrust developed by a constant pressure rocket motor is seen to consist of two parts; the first part can be called the "velocity thrust" F_v and the second part the "pressure thrust" F_p, that is,

$$F = F_v + F_p; \tag{46}$$

also

$$F = mc, \tag{47}$$

where $c = \lambda v_e + \dfrac{(p_e - p_0)f_e}{m}$ = effective exhaust velocity.

The latter form is most generally used, as the value of the effective exhaust velocity c can be easily calculated if the thrust delivered by the motor and the rate of flow of combustibles are known. The thermal efficiency η_{th} expressed by eq. (13) is evaluated in terms of c.

The thrust delivered by an ideal rocket motor unit can now be expressed for the regimes of flow discussed before and illustrated in Fig. 5. For the case in which the area of the

exit is smaller than f_{x_s}* the thrust F_x for a motor whose nozzle is cut at any section $x \geqq x_t$, where x_t is the throat location, is obtained in dimensionless form from eqs. (45), (18), and (22):

$$\frac{F_x}{F_t} = \frac{\lambda\sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}\left[1-\left(\frac{p_x}{p_c}\right)^{(\gamma-1)/\gamma}\right]} + \left(\frac{p_x}{p_c}-\frac{p_0}{p_c}\right)\frac{f_x}{f_t}}{2\left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} - \frac{p_0}{p_c}}, \tag{48}$$

where

$$F_t = \left[2\left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} p_c - p_0\right] f_t \tag{49}$$

is the thrust that would be delivered by the motor unit if the nozzle terminated at the throat, i.e., there is no divergent nozzle part.

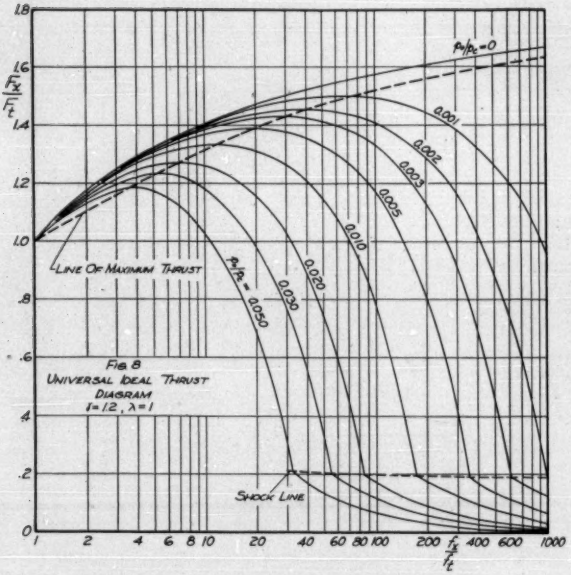
If the exit area is equal or greater than f_{x_s}* then a shock wave will occur and the thrust F_{x_s} is obtained in dimensionless form with respect to F_t from eqs. (45), (18), and (37):

$$\frac{F_{x_s}}{F_t} = \frac{\lambda\gamma}{\gamma-1}\,\frac{-\frac{p_0 f_{x_s}}{p_c f_t} + \sqrt{\left(\frac{p_0 f_{x_s}}{p_c f_t}\right)^2 + 2(\gamma-1)\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}}{2\left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} - \frac{p_0}{p_c}}. \tag{50}$$

In Fig. 8 a universal ideal thrust diagram has been constructed for an ideal rocket motor unit for $\gamma = 1.2$ and for $\lambda = 1$. As the preceding theory predicted, the efficiency and, therefore, the maximum thrust corresponding to complete expansion to p_0 increases as the ratio p_0/p_c decreases. The curve of maximum thrust is indicated by a dotted line.

The fact that over-expansion, if jet separation does not occur, can seriously affect the thrust delivered by the motor, is clearly evident. The possibility of a shock wave occurring in the nozzle is remote unless very low chamber pressures, of the order of 200 to 300 pounds per square inch, are used when p_0 is large, or unless the nozzle is built with much too large an expansion ratio f_x/f_t.

FIG. 8.



Universal ideal thrust diagram for $\gamma = 1.2$ and $\lambda = 1$.

The diagram brings out another important fact. If a nozzle is designed to give the maximum thrust at sea level then the thrust delivered at other altitudes, where the external pressure is much lower, will be less than the amount that can theoretically be obtained. If on the other hand the nozzle is designed to give the maximum possible thrust at a high altitude, then the loss in thrust at lower altitudes may be so great that operation of the rocket will be seriously impaired.

In drawing these conclusions it must be remembered that several simplifying assumptions have been made. It is planned to extend this study of the ideal rocket motor to learn to what extent the effects predicted will be valid for an actual rocket motor.

To illustrate the use of the universal ideal thrust diagram, the characteristics of the following rocket motor unit will be determined:

p_e = 1470 lb./sq. in. abs., d_t = throat diameter = 1 inch,
p_0 = 14.7 lb./sq. in. abs., f_t = 0.786 sq. in.,
p_0/p_e = 0.01, α = 15°,
T_c = 11,100° F abs., λ = 0.985.
γ = 1.2,

Since λ for this example differs from 1 by only a small amount, the diagram in Fig. 8 for $\lambda = 1$ can be used. From Fig. 8 it is found that the maximum thrust along the curve $p_0/p_c = 0.01$ is obtained for an expansion ratio of:

$$\frac{f_x}{f_t} = \frac{f_e}{f_t} = 11.85.$$

The thrust ratio corresponding to this expansion ratio is:

$$\frac{F}{F_t} = 1.338,$$

and eq. 49 gives F_t = 1395 lbs.; therefore,

$$F = 1865 \text{ lbs.}$$

CONCLUSION.

This analysis of the ideal constant pressure rocket motor was made to bring out most of its operating characteristics and to determine the relative importance of various factors that affect its operation. The assumptions introduced greatly reduced the complexity of the mathematical analysis. However, the main features of operation of the actual constant pressure rocket motor will still be the same as for the ideal motor, therefore, the simple general theory presented can be used as a guide by the designer.

The author has been fortunate that during the conduction of this study he has been able to discuss many of its parts with Dr. Theodore de Kármán and especially with Dr. Hsue-shen Tsien. Also, an expression of indebtedness is due to Messrs. J. W. Parsons, A. M. O. Smith, E. Forman, and J. W. Braithwaite—members of the rocket research group of

the Guggenheim Aeronautical Laboratory at the California Institute of Technology.

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A NOVELTY IN MARINE SURVEYING.

BY

C. S. LAWTON,

Western Union Telegraph Company.

During the recent development of the cable plow, a tool for embedding submarine telegraph cable in the ocean bottom for protection against damage by fishing trawlers, the necessity arose for studying irregularities in the bottom surface in much closer detail than is possible with ordinary marine surveying and navigational equipment.

The "echo" or "sonic" method of sounding provides a means for the ready determination of bottom characteristics in much more detail than was possible by the old line and sinker method a few years ago, but like any other method of measuring depth, its accuracy is limited by the vertical motion of the floating body from which the measurements are taken, so that for depth variations on the order of 10 feet, for example, it is quite useless in the open ocean where there is always some swell even in the calmest sea.

The advantage of determining variations in depth of this relatively minute order when geographical positions at sea cannot be fixed to anything like that degree of precision, may not be apparent without some explanation of the plowing procedure.

The cable to be entrenched is paid out from the ship's storage tanks through the plow as the latter is being towed along the bottom behind the ship. Depths of water have ranged from 90 fathoms down to 450 fathoms. As the depth of water increases the towline must be lengthened to avoid lifting the head of the plow off bottom. Thus the length of cable in suspension between the ship and the plow is gradually increasing, until at the deep end there may be as much as a mile of cable between the ship and the plow.

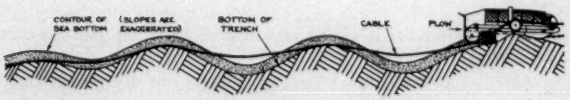
Naturally the tension on the cable as it passes through the plow cannot be held within such fine limits at the deep end as at the shallow end of the work. Lateral sub-surface currents

operate to offset the line of cable from the line travelled by the ship and plow, often making it unsafe to keep the cable tension down to the desired minimum because of danger that the cable will build up slack off to one side of the plow and eventually become foul. Should this happen the only thing to be done is to pick everything up and try again, first cutting out or repairing the damaged cable. Under weather conditions prevailing in the North Atlantic this may easily involve a month's delay or even postponement until another summer season.[1]

From the foregoing it will be seen that the avoidance of some residual tension on the cable placed in the trench is impossible. Therefore it follows that in order to embed the cable at all points along the route, the ground must be free from irregularities where there is a too rapid rate of change of slope. Otherwise there may not be enough weight per unit length of cable to hold it down in the bottom of the trench after the plow has passed the lowest point of a depression or "concavity" in the ground and is climbing up the side. If too much of the required sag be pulled out, the condition illustrated in Fig. 1 is the result. The trench itself can absorb

FIG. 1.

TYPE OF CONTOUR TO BE AVOIDED



only the very minute depressions (as at the left end of the Figure) because the excavated material which eventually closes the trench is too badly disturbed and diluted at the time to press down upon the cable. What happens is rather a silting of material around and underneath the cable, which supports it in whatever position it finally comes to rest.

For this reason, before plowing is undertaken it is essential that the ground be explored with a device capable of indicating

[1] "The Submarine Cable Plow" by C. S. Lawton, presented to the A.I.E.E. at New York, N. Y., January 24, 1939.

not only the presence of these small concavities but also of affording a fair measure of their slopes and extent. Although many of the concavities are too small to be dependably relocated individually, a careful survey of all the ground in the area available for plowing does provide general data for the selection of the best probable route. Then, when the plowing is being done, an accurate record can be made of the slopes of the actual ground traversed, from which it may be definitely determined whether this feature of the job is working out as intended.

As a by-product of the plow development, an electrical instrument has been put into use which when towed across a cable lying on the bottom indicates its exact location. Since the plowing work ordinarily is done in close proximity to an existing cable, it is possible to use the latter as a reference, which means that mark buoys whose geographical position by celestial observation may be out a half a mile or so even under the most favorable conditions, can at least be checked very accurately with respect to one fixed line to which the ship can return at will.

In the first approach to the survey problem it was clear that the desirable arrangement would be a pendulum mounted on a sled which could be dragged along the bottom and which would be electrically connected with the ship, but the connecting member, or members, presented a serious problem. It was desired to use the pendulum on the plow itself. Considering the large working stresses and liability to chafe and other unavoidable abuse inherent in the plow towline and the necessity for frequent readjustment of its length with changes in depth of water, the idea of building and handling a combined towline and electrical cable did not look attractive. On the other hand, the successful handling of an extra line between the plow and ship was a serious problem with working depths ranging down to nearly half a mile.

In the first pendulums constructed, both these difficulties were avoided by placing a recording clockwork with the pendulum in a watertight container designed to withstand the full hydrostatic pressure. The simplicity of the rig was intended to offset the obvious disadvantage of not being able to see the record as it was being made. In order to ensure

that there would be no leakage through the joint around the cover, nitrogen gas was admitted through a valve into the container after the recording pendulum had been placed inside and the cover bolted down. The internal gas pressure was raised to about 150 lbs. per sq. in. and the valve closed. Soapsuds applied around the joint were used to test for leakage. Only after proving the joint tight was the unit submerged. The gas pressure was left in the container, so that upon being lowered away, the external hydrostatic pressure gradually reduced the differential until at a depth of approximately 56 fathoms there was an exact balance between the internal and external pressure, while at greater depths the differential which the container was called upon to withstand was correspondingly reduced. In practice it was found, as anticipated, that the joint if made tight under internal pressure would never leak under external pressure, as this merely tended to press the cover against the copper gasket. The containers, Fig. 2, were in effect small bathyspheres of cast chrome nickel molybdenum steel, 20" inside diameter with walls 7/16" thick. They were successfully used in depths down to 450 fathoms where the hydrostatic pressure is 1200 lbs. per sq. in.

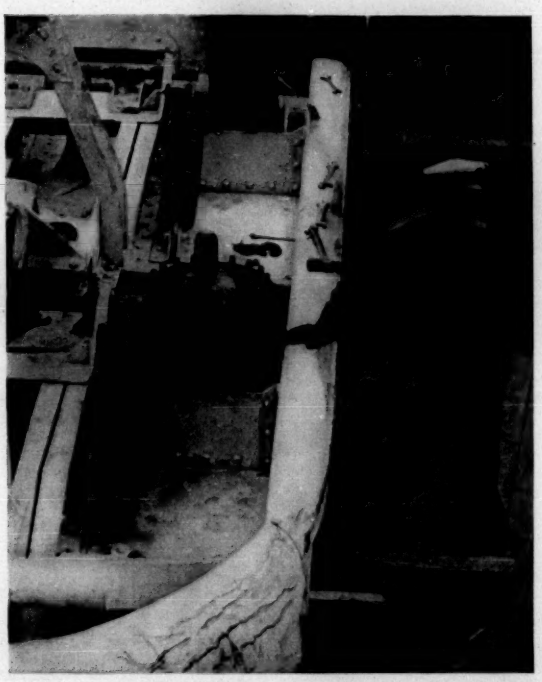
To absorb any moisture in the air inside the container a small quantity of silica gel was carried on the recorder frame in a perforated holder.

The pendulum recorder unit shown in Fig. 3 was constructed in the Electronics Laboratory of The Western Union Telegraph Company at Watermill, Long Island, using an Esterline-Angus clock and paper drive, but substituting an ordinary fountain pen for the usual syphon or ink-well type in order to permit operation in any position and give the required ink capacity for a run of several hours. When the gas pressure inside the container was relieved after the apparatus had been returned to the ship's deck it was unavoidable that most of the remaining ink in the rubber pen sac should be forced out on to the paper, but raising the pendulum from the ocean bottom consumed enough time for the clockwork to wind up the last of the paper record containing the working period, so the ink-smeared portion could

be simply discarded. A special free-flowing ink was used which would not clog the pen.

The pendulum was provided with stops set so as to allow a 45° swing each side of vertical, and with a pneumatic

FIG. 2.

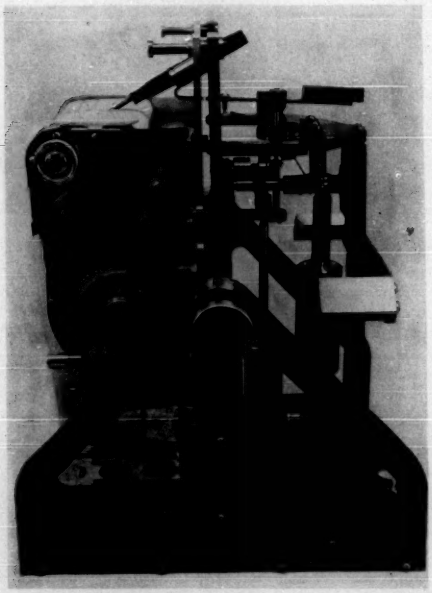


damping arrangement. When allowing a pendulum to swing in the plane of its own progression, unless the speed of progression can be maintained perfectly uniform, inertia effects are bound to produce temporary angular displacements of the pendulum from vertical. The function of the damping

action is to absorb as much as possible of these unwanted manifestations of jerkiness incidental to the towing, while not obscuring any persistent variations in ground slope.

In order to keep the axis of the longitudinal pendulum horizontal when the platform upon which it rested tilted

FIG. 3.



transversely, the whole pendulum recorder assembly was carried on two ball trunnion bearings within the container, set parallel to the plane of the pendulum and well above the center of gravity.

Shortly after these pendulums had been put into service the need became acute for instruments to yield instantaneous information of certain other features of the plow's behavior,

which led to the design and use of a four-conductor "messenger cable" between the plow and ship, each conductor of which would suffice for a single circuit, the sea water being used for the returns. Although doubts had been expressed as to the feasibility of this extra member, when the demand became sufficiently pressing, a method of handling it had to be (and was) gradually evolved through practice.

The obstacle to an instantaneous recording of pendulum data having been removed through the provision of a con-

FIG. 4.

ductor in the messenger cable for this purpose, a new pendulum was designed for use on the plow or for independent survey work on a small sled. (When used on the sled, the towing tension being low, it is easy to arrange a combined towline and communication cable.) In the new unit, Fig. 4, a rheostat is mounted on the pendulum shaft. The necessity for bringing the electrical connection through a gland in the wall of the container indicated a source of trouble if any substantial pressure differential were allowed to build up

between the two sides of the gland. As the recording mechanism had been removed to the bridge of the ship it was decided to fill the container with an insulating oil of low compressibility, incorporating a flexible diaphragm or bellows in the wall of the container to ensure equalization of pressure with such variations in volume as would be induced by temperature and pressure variations. A high-viscosity castor oil was selected which not only satisfied this condition but also provided the required damping for the pendulum. A further beneficial effect was that the container, Fig. 5, could

FIG. 5.

be constructed of thin plate without fear of its being collapsed by the hydrostatic pressure.

Since the directive force of a pendulum diminishes as it approaches the vertical and since we were principally interested in recording small angular displacements, particular pains were taken to reduce the bearing friction while at the same time the weight of the pendulum was substantially

increased. The combined pendulum and rheostat shaft was mounted in ball bearings set in rubber shock-absorbing cushions. The contact arm of the rheostat was provided with a double roller, riding on two rings in which 2° segments were staggered. The result was reliable sensitivity to 1°. The rheostat resistances were wound so as to give equal variations in current readings with equal angular variations of the pendulum up to 15° both sides of 0° slope. Beyond these points the scale is contracted. A variable ballast resistor is mounted near the recording milliammeter on the ship's bridge to compensate for changes in the resistance of the connecting cable with adjustments in the total length in circuit.

In both this and the preceding model of pendulum the 0° position of the recording pen is in the center of the graph paper, so that it is not only possible to read the degree of inclination but also to distinguish which end of the base is up or down, of course.

The graph paper for the electrical recorder is graduated in 10-milliampere steps from 0 to 500. 0° slope is 250 mils. Each degree of slope corresponds to a division of the paper up to 15°, which allows of the use of the instrument maker's standard graph. To indicate the presence of an open or short circuit, the working range between the stops of the pendulum is set between 20 and 480 mils. A reading below or above this range after the pendulum has been correctly adjusted indicates trouble. As the accuracy of the readings depends upon a uniform electric potential, batteries have been found preferable for the power supply.

Figure 6 shows a pendulum-rheostat unit, two of which were built at Western Union's London Repair Shop, mounted on its sled. This assembly together with the recorder and connecting cable is known as a "clinometer." The hoops over the sled cause it to right itself if it lands wrong side up or is thrown over while being towed.

In operating the instrument, care has to be taken to maintain a sufficient length and weight of towline and a sufficiently slow speed to avoid lifting the nose of the sled off bottom. Fortunately when this happens it usually can

be detected, and the operating limits are easily recognized with practice.

After mark buoys have been placed at convenient intervals for ranging, and their positions (relative to the existing cable line and to each other, as well as their geographical positions) have been established, the ground is first gone over quickly with the echo sounder to eliminate areas containing any large scale irregularities which appear dangerous. A more detailed examination is then made of the remaining ground, both with the echo sounder and finally with the clinometer.

FIG. 6.

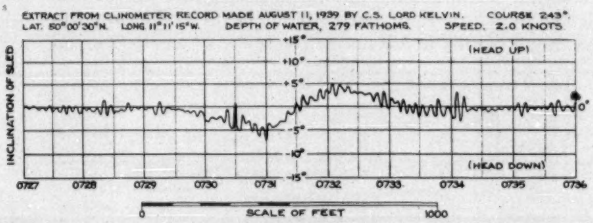
Being interested in finding a route roughly parallel to the existing cable line and as close thereto as practicable, the area under survey generally is a long narrow rectangle, with one end in shallow water and the other in deep. In towing the clinometer sled, it is more practicable to travel parallel to the cable line than normal to it because so much time is lost on short lateral tows in making the necessary "U" turns, particularly at the deep end where, because of the necessity for a much longer towline, the ship has to travel so much farther in towing the clinometer sled back and forth between the parallel sides of the rectangle.

Current set, occasional fog patches and rain squalls

which obscure the mark buoys, wind and other characteristic sea conditions all interfere with the execution of any rigid pattern, while the urgent necessity for completing the survey and doing the plowing before the good weather breaks, usually prevents carrying out the work with the degree of thoroughness desired. Nevertheless the results obtained have amply demonstrated the value of the instrument for this class of work when in the hands of an experienced cable man like the late Captain M. H. Bloomer, who was Commander of the Western Union Cable Ship "Lord Kelvin" during this work.

A typical bottom concavity registered with this pendulum is shown in Fig. 7, in which the irrelevant portion of the

FIG. 7.



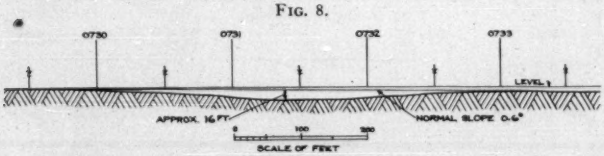
graph paper has been omitted and main divisions redesignated in degrees of slope. The vertical divisions give Greenwich Mean Time in minutes. The small angular variations of from 1° to 3° occurring with a frequency of 1/8 minute or less are typical inertia effects caused by the rise and fall of the ship on the swell, with which motion they are always observed to synchronize. The true ground slope is taken as a mean drawn through them, but due to the damping employed they are so small in extent that even if plotted accurately to scale with the larger variation upon which they are superimposed it would take a very sharp eye to detect them in a profile chart.

It will be obvious that the first step in converting the pendulum graph into a profile of the ground is to convert the time ordinate into distance, using the speed of the ship. The latter is determined from frequent ranges and bearings

taken on mark buoys, and except when low visibility obscures these it is fairly dependable. When this has been done the slope curve is ready for integration into the profile chart shown in Fig. 8.

To assess the hazard introduced by a concavity of this kind when trenching cable, a series of templates are prepared to the same scale as the profile chart, each representing a catenary curve in sea water of the cable intended to be entrenched, but under various tensions at the vertex. By keeping the axes of the catenaries vertical and comparing them with the profile of the concavity it is a simple matter to determine the maximum allowable residual tension for the cable if suspension is to be avoided. As previously mentioned, a certain small amount of suspension can be absorbed in the depth of the trench, but it is preferable not to count on much of this in advance.

FIG. 8.

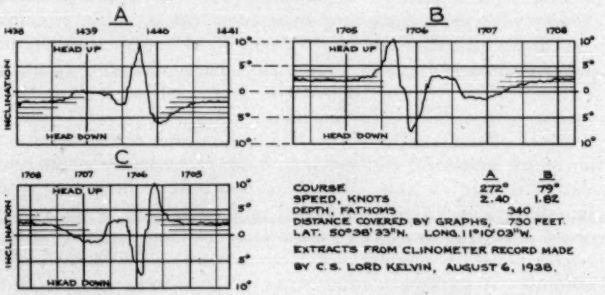


During surveying operations in the summer of 1938 the clinometer passed over a peculiar irregularity which it seemed desirable to re-locate if possible. The record is shown in Fig. 9 at "*A*." After proceeding some distance beyond, the ship was swung around and the sled was taken back as nearly as possible over the same route. When the latter reached the same position about an hour and a half later, the record shown at "*B*" was obtained. The ship had been purposely slowed down to make doubly sure, which accounts for the second record being longer than the first. To compare the two records, the second has been contracted to the same distance scale as the first, then reversed end for end at "*C*." Bearing in mind that the sled traversed nearly parallel routes but that in all probability it did not reënter the concavity at exactly the same spot, the similarity in the records is striking. It will be noted that the prevailing normal ground slope was

about 2° to 2½°, the instrument zero coinciding within its limiting accuracy of one degree as shown by the reversal of slope in the two records.

It is interesting to note that some of the preconceived notions of ground slopes on the ocean bed have been modified considerably by the more intensive and detailed surveys undertaken in connection with cable and other marine work in recent years. It is generally appreciated among those seriously interested that ocean profile charts dished up for public consumption give a totally erroneous impression of the slopes, as not only is it customary to use different scales for the vertical and horizontal ordinates, but it is not un-

FIG. 9.



common for the vertical scale to be several hundred times the horizontal. If the ordinates were plotted in true relationship, most of these charts would be flat and uninteresting, which is indeed the case with most of the ocean bed.

Gradients of more than 1 in 10 are rare over any great distance. The relatively quick drop off the edge of the continental "shelf" is well within this gradient. Recently the U. S. Coast & Geodetic Survey have published data concerning "canyons" cut through the edge of the shelf which have astonishingly steep gradients on the order of 1 in 3, but even these if plotted to scale would resemble depressions in a rolling countryside more than they would the conventional canyon.

Whereas gradients greater than 1 in 10 are not usually detected by old-fashioned sounding methods because of their limited extent and the limited number of soundings which could be taken in the time available, there appear to be many instances where larger gradients are common over short distances. For instance, while surveying with an echo sounder last summer in the vicinity of the position given above for the data in Fig. 6, gradients were repeatedly found as great as 1 in 6.3 and there was one indication of a gradient of 1 in 4.3. Unfortunately the latter could be only approximated as the speed of the ship was not accurately known, fog having temporarily obscured the mark buoy.

Passing now to the very small irregularities in the ocean bottom which perhaps are of no practical significance at present except in trenching submarine cable, again we find very few gradients in excess of 1 in 10. Here, of course, our primary interest in cable laying lies in the second derivative of the profile curve, or the rate of change of slope since that is what really creates the hazard.

Most of the bottom which has come under observation off the south coast of Ireland has been remarkably even and nearly as flat as the proverbial pancake. The plow and clinometer sled have been towed mile after mile without the record deviating more than a few degrees from the prevailing ground slope in passing from 90 fathoms of water to 450 fathoms. A striking confirmation of the smoothness of some bottom was obtained during experiments with the 9-ton plow in 1936 when on one occasion the share, or member which digs the trench, had been temporarily removed to obtain some idea of the coefficient of sliding friction of the plow runners. The plow which is 20 feet long is supported on four of these runners; two forward and two aft, in line. They each have a bearing surface $3\frac{1}{2}$ feet long and at that time were 8 inches wide and well-rusted where the paint had worn away. The plow was lowered on a hard sand bottom and towed 1.3 nautical miles. Upon recovery the bottoms of the runners were found to have been brightly polished but at the forward ends where they were bent up at an angle of 30° the paint and rust remained undisturbed down to within 1″ vertically from the horizontal bearing surfaces on all four

runners. Our experience with the clinometer sled has been similar.

It will thus be seen that while the greatest slopes encountered off Ireland were too limited in extent to be found by the old line and sinker method, they did extend over fairly large distances as we are acustomed to think of distances on land. Variations small enough to be bridged by the clinometer sled which has a flat bottom six feet long, are of course not considered, except in special cases where the limiting length was that of the plow runners.

Practically all the ground referred to is gray sand, the grains becoming finer as the water gets deeper beyond 100 fathoms until gradually it changes to mud in the region of 300 to 400 fathoms. It is very hard-packed in 100 fathoms but gets softer as the grains become finer. In other words it makes excellent fishing grounds, as might be expected.

Colloidal Carbon as a Grinding Aid in Portland Cement Manufacture.—Colloidal carbon, commonly known as carbon black, is an effective grinding aid in the manufacture of portland cement, according to the results of research carried out by the Columbian Carbon Company's Industrial Fellowship at Mellon Instutute, Pittsburgh. The report of this investigation discloses that a carbon dosage as low as 0.32 per cent. on the clinker increases the fineness of the cement by 30 per cent. when the time of grinding is constant, and that the same carbon dosage decreases the grinding time by 28 per cent. when the grinding is run to constant fineness. With a 1 per cent. dosage these improvements become 50 per cent. and 34 per cent., respectively. In terms of power saving and increased output these results are of practical significance. The cements prepared with carbon present as a grinding aid compared to the controls showed improved strength properties in tensile and compression tests on mortars. These benefits are attributed in the main to the increased fineness of the cement, although there is evidence that the carbon per se is contributory. The use of carbon in dosages up to 1 per cent. does not alter appreciably such standard properties of portland cement as consistency, setting time, and soundness, color excepted, and has no noticeable effect on the reistance of cement mortars to freezing and thawing treatment. The effectiveness of colloidal carbon as a grinding aid, together with the existence of a reinforcing effect, is thought to be established by these experiments. A new application for colloidal carbon, which has already reached important dimensions, is thus indicated for the cement and related industries. The colloidal carbon used throughout this study was the grade termed "Cem Beads," a pellet form of a standard channel carbon manufactured from natural gas, with physical and chemical properties essentially similar to the colloidal carbon pigment used in the rubber industry. The diameter of the ultimate carbon particles in the pellet averages about 30 millimicrons (0.000001 inch), thus establishing the colloidal nature of the pigment. Copies of a publication descriptive of the research may be obtained by writing to Mellon Institute.

R. H. O.

COMPRESSED MAGNESIA AS ELECTRICAL INSULATOR.

BY

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ABSTRACT.

Compressed magnesia exhibits a fairly high breakdown strength of 300 kv./cm. This high strength appears remarkable, in considering the large porosity factor of the wafers. The contradiction can be explained by the inorganic nature of the material, and its consequent resistance against overheating and ionization.

1. INTRODUCTION.

Recently the authors have investigated the electrical properties of fibrous glass,[1] this new insulating material, the qualities of which furnish a striking proof for the superiority of inorganic insulators as compared with organic ones. Unfortunately there does not up to the present exist a plastic, homogeneous inorganic dielectric material, however desirable it would be both in combination with fiber glass and alone.

In this connection the authors have noted a paper of M. J. Bethenod,[2] describing the so-called Pyrotenax cable with compressed magnesia insulation, and mentioning that this dielectric, if subjected to deformation, behaves as a homogeneous, plastic material. Although the analogy to a plastic is evidently only external, it was thought worth while to further investigate this insulator, first of all its electric strength, the more so, as there are no quantitative data in this respect available in Bethenod's paper.

As the chief advantage of magnesia is its incombustibility, the cables just mentioned, like the heating units, known as Calrod, made with magnesia, are intended for high currents, and not for high voltages. Still, it might be useful to look upon this material as an insulator for reasonably high voltages, and investigate its behavior in this respect.

* Present address: Detroit Edison Company, Detroit, Michigan.

[1] A. Gemant and F. A. Glassow, *Electrical Engineering*, *58*, 341, 1939.

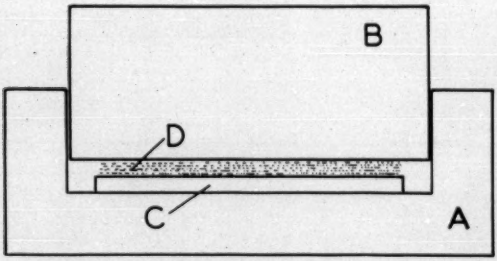
[2] M. J. Bethenod, *Bull. Soc. Franç. Elect.*, *8*, 325, 1938.

The results of this work are contained in the following paper. Apart from measurements of the electric strength, some data on its power factor and its porosity will be included.

2. ELECTRIC STRENGTH.

The material used for making compressed samples was the "light powder" variety of magnesium oxide. This was brought into a steel chamber shown in Fig. 1; *A* is the con-

FIG. 1.



Pressure chamber for making samples of compressed magnesia.

tainer, *B* a cylindrical piston. A steel disc *C*, covered with tin foil, was placed into the chamber, then the powder *D*, followed by a sheet of paper and finally the piston. The paper prevented sticking of the layer to the piston.

The whole device was brought into a small hydraulic press, capable of producing 50,000 lbs. pressure. Since the approximate surface of the sample was 7.5 sq. in., the pressure used in these experiments was about 6,500 lbs. per sq. in. When finished, the disc *C* was lifted from the container, and separated from the tin foil to which the finished magnesia layer adhered. Such wafers were then used for the breakdown tests.

The tin foil (1.5 mils thick) served simultaneously as the lower electrode and mechanical support. This latter function was fairly important, as compressed layers of this type did not exhibit any "plasticity," but were rather brittle. The thickness of the samples was of the order of 0.2 mm.

V. 19

With regard to the technique of the puncture tests the following should be mentioned. The obtained electric strength was fairly high, as will be seen below. Consequently the puncture was preceded by a corona discharge from the top electrode, a brass sphere of 1 inch diameter. It is possible that the measured puncture values were affected somewhat by this preliminary corona, and it was thought advisable to eliminate it. The easiest way to do this, namely, submerging of the whole assembly into oil was, however, not feasible in this case. Because of the porosity of the samples, of which more will be said below, oil would probably penetrate into the interior, thus obviating the results. Accordingly, measurements were carried out in air, in spite of the corona, and, in order to minimize the discharge, only the central portions of the circular samples were tested.

An ideal, well-known arrangement for such materials is to make the whole specimen appreciably thicker than its central portion which, in form of a flat cavity, closely follows the shape of the electrode. This, however, would require a more complicated moulding device instead of that shown in Fig. 1.

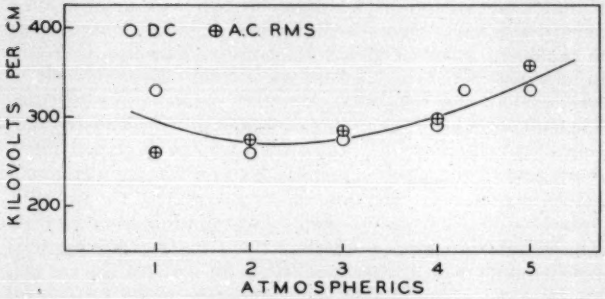
Puncture tests were carried out with both alternating and continuous voltages. The thickness of the specimens was measured by means of a micrometer, that of the tin foil being deducted from the total. Results were then computed as breakdown strength in kilovolts per cm., data for a.c. voltages being r.m.s. values.

Figure 2 shows the results of our tests for different air pressures, obtained by means of a special pressure chamber and compressed nitrogen gas. The following conclusions can be drawn:

1. The absolute value of the strength is around 300 kv./cm., a value not far behind that of porcelain (350 kv./cm.). This is surprising, considering the high porosity of the material. (See below.)

2. D.c. and effective a.c. values are practically the same, indicating that the final puncture is not essentially caused by an ionization in the air pockets, but that it is chiefly a thermal failure.

FIG. 2.

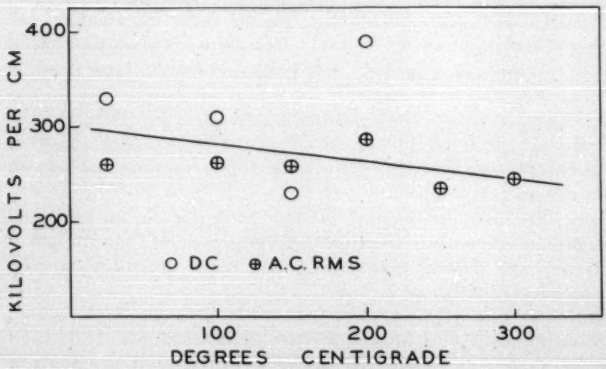


Electric strength as function of air pressure.

3. As to the variation with pressure, there is, apart from a flat minimum in the first portion of the curve, a slight increase with pressure of the electric strength. The gain up to 5 atmospherics is about 20 per cent.

Figure 3 indicates the electric strength, plotted as a

FIG. 3.



Electric strength as function of temperature.

function of temperature. Measurements were carried out in a small oven with a high tension bushing. D.c. tests were difficult to make because of the frequent flashover discharges, probably due to surface creepage. There appears to be a slight decrease with temperature of the electric strength, but not more than about 15 per cent. up to 300° C.

Each point in Figs. 2 and 3 is the average from a large number of individual data which were scattered considerably around the average. This is to be expected with this type of material, since the density is not uniform, but varies from point to point. Even the average d.c. data on Fig. 3 show a rather irregular behavior.

Altogether the results seem to indicate that compressed magnesia could be used for the insulation of fairly high voltages. Only its brittleness ought to be reduced either by altering the structure and size of the grain or by a suitable admixture.

In addition a few short time tests with a.c. voltages are shown in Table 1. It can be seen that the drop from the

TABLE 1.

Short Time Puncture Tests.

Pressure in Atmos.	Time in sec.	Breakdown Strength in kv./cm.
1	< 1	295
1	>300	280
3	< 1	280
3	15	285
3	35	265
3	150	270
3	>300	265

immediate puncture value to that which causes no failure within 5 minutes is small, about 5 per cent. Long time tests have not been carried out.

In order to have an idea whether a further improvement of the strength beyond that already obtained is possible, similar samples of the "heavy powder" variety of MgO were prepared. Because of its smaller compressibility the wafers turned out to be about 0.4 mm. thick. Some tests are summarized in Table 2. The values are considerably lower

TABLE 2.

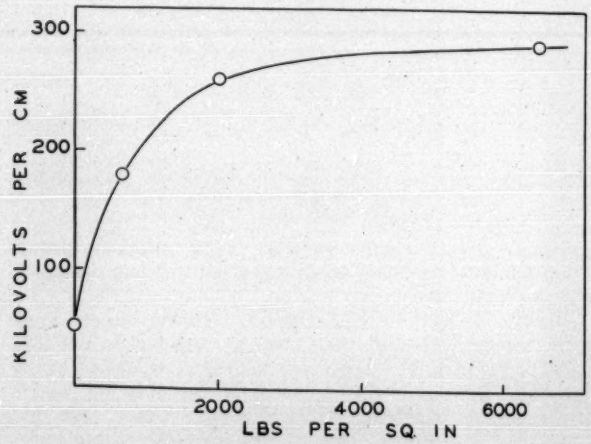
Data Obtained from "Heavy Powder" Samples.

Temperature in Degrees Centigrade.	Breakdown Strength in kv. (r.m.s.)/cm.
25	116
100	95
150	114
250	127

than the previous ones. Even if this is due partly to the increased thickness of the specimens, it is caused essentially by the coarser grain and wider interstices. Thus there is the possibility that still higher values than those in Figs. 2 and 3 can be obtained by using a magnesia powder of a finer grain than that commercially available to-day.

Another set of measurements was directed towards finding out the role of the pressure during the making of specimens. The results are shown in Fig. 4 where the r.m.s. puncture values are plotted against the moulding pressure. For zero pressure the strength of air for a gap of 0.2 mm., namely 52 kv./cm. was taken.

FIG. 4.



Electric strength as function of moulding pressure.

It can be seen that the strength rises rapidly up to 2,000 lbs. per sq. in., after which the increase is slow. However, by increasing the pressure one order higher, say up to 50,000 lbs. per sq. in., a further improvement of the strength is to be expected.

3. POWER FACTOR.

It is interesting to note that an electric strength of 300 kv./cm. is obtained with compressed magnesia, although the material contains appreciable moisture, if not conditioned at higher temperatures. The temperature curve Fig. 3 also indicates that the water content cannot have great influence upon the strength, as the moisture must have been fairly reduced at 300° C. without improving the electric strength.

This unexpected result can only be explained by considering the purely inorganic nature of the material in question. The conduction component of the current is not in itself as harmful as frequently supposed, only when local overheating leads to oxidation and carbonization, as in the presence of most organic constituents. This view is in accord with that recently expressed by F. M. Clark.[3] If the substance is as resistant to heat as magnesia, then a high conductivity, up to a certain voltage at least, does not seem to be very dangerous. Beyond this voltage limit, of course, thermal instability will set in. Thus the superiority in general of inorganic insulators becomes evident.

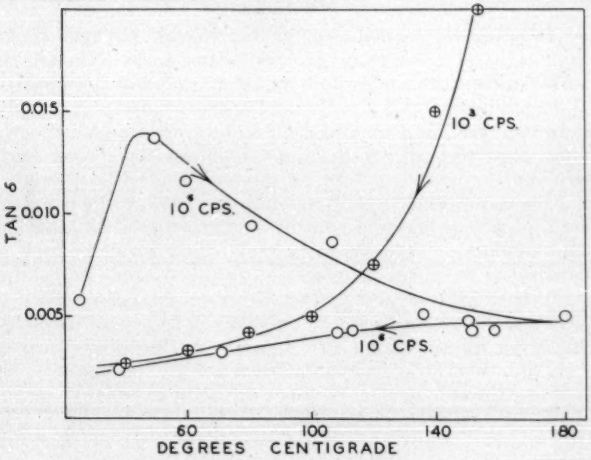
In order to have quantitative data related to the moisture content of the material, power factor measurements were carried out up to temperatures of 180 degrees centigrade. A radio frequency bridge for 10^6 cycles and an audio frequency bridge for 10^3 cycles were used. A 5 cm. diameter brass plate without guard ring served as upper, the tin foil as lower electrode. The result is plotted in Fig. 5.

With 10^6 cycles the initial power factor at room temperature is about 0.006, rising rapidly to 0.014 at 50° C. because of the exponential increase of conductivity due to moisture. From 50° C. onward the power factor drops steadily to 0.005 at 180°, because of the continuous evaporation of water. In returning to room temperature the power factor further

[3] F. M. Clark, *Electrical Engineering*, 57, 489, 1938.

slowly drops to 0.0025. This latter portion of the curve is scarcely affected by the remaining humidity, and exhibits the inherent dielectric losses of the material.

FIG. 5.



Power factor as function of temperature.

With 10^3 cycles the power factor at room temperature of the unconditioned wafer is of the order of unity, thus 10^3 times higher than the difference between the initial and final values at 10^6 cycles. This is to be expected, since the power factor due to conductivity is inversely proportional to the frequency. Because of evaporation of the water it drops to 0.13 at 80° C., and at 160° C. it reaches 0.022. Returning from here to room temperature, the curve shown in Fig. 5 is obtained. From 100° C. down to room temperature the d.c. conductivity drops to such low value that it does not any more affect the 10^3 cycles curve and the measured power factor is nearly the same for both curves, the inherent losses being fairly independent of the frequency.[4]

[4] A. Gemant, *Journ. Applied Physics*, *10*, 508, 1939.

The conclusions, therefore, are:

1. By heating compressed magnesia up to 160° C., the water content is practically removed, and, if prevented from contact with air, a fairly low power factor of around 0.002 is obtained. Such a conditioning is, of course, necessary in any case of practical application of the material.

2. If unconditioned, the moisture content is fairly high, causing large power factors at acoustical and power (but not at radio frequencies). In spite of this the electric strength of the material is reasonably high, an explanation being suggested above in this section.

It might be added that the measured capacity data, together with the knowledge of the thickness of the specimens allowed the dielectric constants to be computed: 2.5 was obtained for 10^6, 2.2 for 10^3 cycles.

4. POROSITY.

Considering the high electric strength of 300 kv./cm., it seemed desirable to obtain quantitative data on the porosity of the samples.

The following measuring procedure was adopted. A piece of tin foil completely covered with compressed powder was weighed and its thickness determined by means of a micrometer. Deducting 1.5 mils from this latter, the net thickness, d, was obtained. Removing the powder from the foil and weighing the latter, the net weight, w, of the powder was known. Knowing the specific surface weight (0.0275 gr./sq. cm.) of the tin foil, the surface, f, of the wafer can be calculated. Thus the total volume, v, is given by

$$v = fd.$$

The actual volume, v_0, of the solid grains in the layer, on the other hand, is

$$v_0 = \frac{w}{\rho}$$

if ρ is the density of magnesia. Thus the porosity in per cents, p, is given by

$$p = 100\,\frac{v - v_0}{v} = 100\left(1 - \frac{w}{\rho fd}\right).$$

As for the density, ρ, its value in the literature varies around 3.4. To check this value, 0.40 gr. MgO were placed into a 5 cc. graduated cylinder, containing a fluid, and the rise in level observed. This was 0.12 cc. for xylene, 0.135 cc. for benzene and 0.145 cc. for water, giving an average density 3.0 for the powder. Thus the average value 3.2 was used.

The results of the measurements are summarized in Table 3. The samples have the large porosity factor of 67

TABLE 3.

Porosity of Compressed Magnesia.

Number of Experiment.	Net Weight in Grams = w.	Surface in sq. cm. = f.	Net Thickness in cm. = d.	Porosity in per cent. = p.
1	0.925	44.0	0.0208	68
2	1.015	38.0	0.0245	66

per cent. A low breakdown strength might, therefore, be expected, not much higher than that for a corresponding air gap.

An explanation for the high strength can be given by first considering that the voids do not allow a direct communication between the two electrodes. Because of the strong moulding pressure the flake shaped particles thoroughly enclose the remaining pockets, thus limiting the ionization in the voids to local processes not leading to an actual breakdown.

Besides, the width of the voids between adjacent particles is certainly small (see below). As the ionizing field strength increases with decreasing air gap, a high value for the strength can be accounted for on this basis.

Nevertheless, ionization certainly takes place within the material before breakdown occurs. It appears, however, that this ionization, if moderate, does not affect the solid framework, if the latter consists of inorganic material. This might be the reason that compressed magnesia has a much higher electric strength than even strongly calendered cable paper of the same porosity factor.

It is possible to estimate the width of the air pockets on the above basis. Assuming a series arrangement of magnesium oxide flakes and air pockets, and a dielectric constant

of, say, 6 for the solid, we have for the two fields F_a and F_m in the air and in the magnesia near breakdown the two equations:

$$0.67F_a + 0.33F_m = 300,$$
$$F_a = 6F_m,$$

giving $F_a = 420$ kv./cm. This ionization field is consistent with a width of $\sim 10\mu$, since the sparking potential at that separation is around the so-called minimum voltage, namely 400 volts,[5] giving a field of $400/10^{-3}$ v/cm., that is just the right order for F_a.

SUMMARY.

In applying a pressure of about 6,500 lbs. per sq. in., wafers of compressed magnesium oxide can be made, the electric strength of which both for a.c. and d.c. voltages is about 300 kv./cm. This value shows a slight increase with increasing air pressure up to 5 atmospheres, and a slight decrease with increasing temperature up to 300° Centigrade.

The material, if unconditioned, has a certain moisture content which for lower frequencies causes a large power factor. If heated up to 160° C., the moisture is removed, and the power factor at room temperature drops to about 0.002 for both lower and radio frequencies.

Besides, the compressed powder has a high porosity. Determination of the latter revealed a porosity factor of around 67 per cent.

The electric strength even of the unconditioned material is, in spite of these two unfavorable features, comparatively high; this can be explained by the fact that the samples, because of their inorganic nature, are strongly resistant against overheating, due to conduction currents, and local ionization which certainly occurs at higher field strengths.

The results, therefore, furnish a further proof of the superiority in general of inorganic insulating materials.

[5] Knoll-Ollendorff, "Gasentladungstabellen," Springer, Berlin, 1935, p. 84.

Relation of the Tide to Property Boundaries.—The U. S. Coast and Geodetic Survey has recently published a pamphlet by the late Rear Admiral R. S. Patton which treats briefly, and with as little technical detail as possible, the physical factors which must serve as a background to any consideration of this subject. The grants, charters and conveyances which constitute the first links in the chains of title on which are based the present ownerships of lands along our seacoasts contain frequent reference to such boundaries as the high water line, the high tide line, the line of ordinary high water, etc., and similar reference to the opposite, or low stage of the tide. As a rule these references are indefinite to the point of ambiguity, primarily because of the inherently complex and variable character of the tidal phenomena, and secondarily because, at the time the early descriptions were written, either the significance of the first factor was not appreciated, or it was not considered of sufficient importance to require precise definitions of the phrases used. The result is that our courts are called upon from time to time for precise and workable interpretations of these vague and ambiguous phrases. The frequency with which such interpretations are necessary is strikingly evident to the Coast and Geodetic Survey, which, as the agency of the Federal government officially charged with the study and prediction of the tides, and the generally recognized authority in this country on that subject, is called upon many times each year for tabulations of tidal data applicable to this or that matter in litigation. Statistical data, however, serve the ends of justice only to the extent that they are properly interpreted, and proper interpretation of tidal data is usually possible only when the tabulation is appraised against a background of pertinent scientific principles and physical facts pertaining to the situation under consideration.

R. H. O.

A MATRIX GENERALIZATION OF HEAVISIDE'S EXPANSION THEOREM.

BY

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I. ABSTRACT.

Starting with the energy and dissipation functions of the general n mesh linear bilateral network and using the operational methods of the Laplacian transformation, a solution is obtained for the Lagrangian equations of the system subject to initial boundary conditions. The equations take a particularly simple and general form if matrix notation is used. It is noted that the general case bears a close resemblance to the simple, one mesh, series circuit when the scalar factors which appear in this circuit are generalized to matrix form.

II. INTRODUCTION.

There is probably no more famous application of the operational method than Heaviside's "expansion theorem." It has been extensively used in the field of electrical engineering and has been greatly discussed and extended (1–6). It is the purpose of this discussion to derive a general expression for the various currents in a general n mesh electrical system by the use of the Laplacian transformation and the concise notation of matrix algebra. The expression so derived takes into account initial charges and currents and the application of arbitrary electromotive forces at $t = 0$. Since the conventional form of Heaviside's expansion formula does not take into account initial disturbances and is restricted to the application of electromotive forces of the "unit function" type, it thus appears that the method here presented is more general.

III. THE ENERGY FUNCTIONS.

The discussion will be based on Maxwell's dynamical theory and the theory of elementary matrix algebra such as that presented in reference (7) will be assumed. In accordance with Maxwell's method, the network forms a dynamical system in which the currents play the role of velocities. We shall therefore, begin our discussion by a

consideration of the expressions for the kinetic or magnetic energy, the potential or electric energy, and the dissipation function of the system. The network equations are then deducible from these functions by the general dynamical equations of Lagrange.

The general n mesh network is characterized by the following energy functions:

$$T = \{\dot{q}\}'[L]\{\dot{q}\}/2, \tag{1}$$

$$V = \{q\}'[S]\{q\}/2. \tag{2}$$

Here T represents the total magnetic energy of the system and V the total electric energy of the system. $\{q\}$ and $\{\dot{q}\}$ are columnar matrices whose elements are the various mesh charges and mesh currents of the system respectively. $\{q\}'$ and $\{\dot{q}\}'$ are line matrices that are the transposed matrices of $\{q\}$ and $\{\dot{q}\}$.

$[L]$ is a square matrix whose elements are the various mesh and mutual inductances of the network. $[S]$ is a square matrix whose elements are the mesh and mutual elastances of the system. The system is also characterized by a dissipation function:

$$F = \{\dot{q}\}'[R]\{\dot{q}\}/2. \tag{3}$$

F denotes *half* the total instantaneous rate of energy loss. $[R]$ is a square matrix whose elements are the various mesh and mutual resistance coefficients of the system. If the network is composed of n linearly independent meshes, the order of the matrices $[L]$, $[S]$, and $[R]$ is n.

The quadratic forms T, V, and F are characterized by the fact that they are positive for all values of the variables $\{q\}$ and $\{\dot{q}\}$.

IV. THE LAGRANGIAN EQUATIONS.

If the circuit has impressed on its n meshes the n mesh electromotive forces, then we may write the Lagrangian equations of the system in the following form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) + \frac{\partial F}{\partial \dot{q}_k} + \frac{\partial V}{\partial q_k} = e_k(t), \tag{4}$$

where $k = 1, 2, \cdots n$.

Performing the indicated differentiation of the quadratic forms T, F, and V, we obtain a set of equations that may be conveniently written in the following matrix form:

$$[L]\{\ddot{q}\} + [R]\{\dot{q}\} + [S]\{q\} = \{e(t)\}, \tag{5}$$

where the columnar matrix $\{e(t)\}$ has for its elements the various electromotive forces impressed on the corresponding meshes of the system. These are the canonical differential equations of the general linear electrical system in matrix notation.

V. THE GENERAL SINGLE MESH CASE.

For simplicity, let us consider the general one mesh case. The canonical equations (5) reduce to the single scalar equation:

$$L\ddot{q} + R\dot{q} + Sq = e(t), \tag{6}$$

Where L, R, and S are the inductance, resistance, and elastance coefficients respectively and $e(t)$ is the impressed electromotive force. If we let:

$$q = \int_{-\infty}^{t} i dt, \tag{7}$$

we may write this equation in the following manner:

$$Ldi/dt + Ri + S\int_{-\infty}^{t} i dt = e(t). \tag{8}$$

Let us now consider that the electromotive force is applied at $t = 0$ and that at this time the condenser has an initial charge q_0 and the circuit has an initial current i_0. If we introduce $I(p)$ the Laplacian transform of $i(t)$, we then have:

$$I(p) = p\int_{0}^{\infty} e^{-pt} i(t) dt. \tag{9}$$

Similarly, let us introduce $E(p)$, the Laplacian transform of the electromotive force by the equation:

$$E(p) = p\int_{-0}^{\infty} e^{-pt} e(t) dt. \tag{10}$$

In (9) and (10) the real part of p is positive. For brevity, if we have two functions $h(t)$ and $g(p)$ related by a Laplacian transformation in the form:

$$g(p) = p\int_0^\infty e^{-pt}h(t)dt, \tag{11}$$

we shall use the notation:

$$g(p) \doteqdot h(t). \tag{12}$$

Now we have the well-known relations that if,

$$I(p) \doteqdot i(t) \tag{13}$$

then

$$pI - pi_0 \doteqdot di/dt \qquad \text{where} \qquad i_0 = i(0), \tag{14}$$

$$I(p)/p + q_0 \doteqdot \int_{-\infty}^{t} idt \qquad \text{where} \qquad q_0 = \int_{-\infty}^{0} idt. \tag{15}$$

By applying the Laplacian transformation to equation (6) we obtain the following equation to be satisfied by the transform $I(p)$.

$$I(p) = \frac{Lp^2i_0 - Spq_0 + pE(p)}{f(p)}, \tag{16}$$

where

$$f(p) = pZ(p) \tag{17}$$

and

$$Z(p) = Lp + R + S/p. \tag{18}$$

The computation of $i(t)$ from its transform $I(p)$ is effected by the general inversion formula of the Laplacian transformation:

$$i(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{e^{pt}I(p)dp}{p}, \tag{19}$$

where the line integral is taken in the complex p-plane to the right of the singularities of the integrand. The evaluation of this integral is made by the use of the residue calculus and depends on the nature of the singularities of the integrand.

1. *Case.* Distinct roots of $f(p) = 0$.

Using the theory of residues to evaluate the integral (19) we have:

$$(20) \qquad Lp^2 i_0/f(p) \doteq \sum \text{Residues of } Lpi_0 e^{pt}/f(p).$$

Now

$$(21) \qquad \begin{aligned} f(p) = pZ(p) &= Lp^2 + Rp + S \\ &= L(p - p_1)(p - p_2), \end{aligned}$$

where p_1 and p_2 are the roots of $f(p) = 0$. Hence:

$$(22) \qquad Lp^2 i_0/f(p) \doteq \sum \text{Residues of } Lpi_0 e^{pt}/L(p - p_1)(p - p_2)$$

$$\doteq Li_0 \sum_{n=1}^{n=2} p_n e^{p_n t}/f'(p_n),$$

where $f'(p_n) = df/dp$ at $p = p_n$. Similarly:

$$(23) \qquad Spq_0/f(p) \doteq \sum \text{Residues of } Sq_0 e^{pt}/L(p - p_1)(p - p_2)$$

$$\doteq Sq_0 \sum_{n=1}^{n=2} e^{p_n t}/f'(p_n)$$

We must now compute the inverse transform of the expression:

$$(24) \qquad pE(p)/f(p) = pE(p)/L(p - p_1)(p - p_2).$$

To do this, let us expand $p/f(p)$ into partial fractions in the following manner:

$$(25) \qquad \frac{p}{L(p - p_1)(p - p_2)} = A_1/(p - p_1) + A_2/(p - p_2),$$

we thus obtain:

$$(26) \qquad A_1 = p_1/f'(p_1), \qquad A_2 = p_2/f'(p_2).$$

Hence:

$$(27) \qquad pE(p)/f(p) = A_1 E(p)/(p - p_1) + A_2 E(p)/(p - p_2).$$

To compute the transform of $E(p)/(p - p_1)$ we make use of one of the fundamental theorems of the operational calculus, the Faltung theorem which may be stated in the following manner. If:

$$(29) \qquad g_1(p) \doteq h_1(t) \qquad \text{and} \qquad g_2(p) \doteq h_2(t),$$

then

$$g_1g_2/p \doteqdot \int_0^t h_1(u)h_2(t-u)du. \tag{30}$$

In this case, let

$$g_1(p) = E(p) \doteqdot e(t) \tag{31}$$

and

$$g_2(p) = p/(p - p_1) \doteqdot e^{p_1 t}. \tag{32}$$

Hence:

$$E(p)/(p - p_1) = g_1g_2/p \doteqdot \int_0^t e^{p_1(t-u)}e(u)du. \tag{33}$$

We thus obtain:

$$pE(p)/f(p) \doteqdot \sum_{n=1}^{n=2} \frac{p_n}{f'(p_n)} \int_0^t e^{p_n(t-u)}e(u)du. \tag{34}$$

We thus have for the complete solution in the case of distinct roots of the expression $f(p) = 0$, the following equation:

$$i(t) = Li_0 \sum_{n=1}^{n=2} p_n e^{p_n t}/f'(p_n) - Sq_0 \sum_{n=1}^{n=2} e^{p_n t}/f'(p_n) + \sum_{n=1}^{n=2} \frac{p_n}{f'(p_n)} \int_0^t e^{p_n(t-u)}e(u)du. \tag{35}$$

This is a particularly convenient form for the case of a general electromotive force $e(t)$ applied at $t = 0$. The effect of the initial conditions is clearly evident.

2. *Case. Repeated roots of* $f(p) = 0$.

In case that $f(p) = 0$ has repeated roots, we may write:

$$f(p) = L(p - p_1)^2. \tag{36}$$

In such a case we have:

$$Lp^2i_0/f(p) \doteqdot \sum \text{Residues of } Lpi_0e^{pt}/(p - p_1)^2 \doteqdot i_0e^{p_1t}(1 + p_1t). \tag{37}$$

Similarly:

$$Spq_0/f(p) \doteqdot \sum \text{Residues of } Sq_0e^{pt}/L(p - p_1)^2 \doteqdot q_0Ste^{p_1t}/L. \tag{38}$$

To compute the inverse transform of the following expression,

$$pE(p)/f(p) = pE(p)/L(p - p_1)^2, \tag{39}$$

we again use the Faltung theorem (29). Let

$$g_1(p) = E(p) \doteqdot e(t) \tag{40}$$

$$g_2(p) = p^2/(p - p_1)^2 \doteqdot e^{p_1 t}(1 + p_1 t).$$

Therefore:

$$pE(p)/(p - p_1)^2 \doteqdot \int_0^t e^{p_1(t-u)}\{1 + p_1(t - u)\}e(u)du. \tag{41}$$

We then have:

$$i(t) = i_0 e^{p_1 t}(1 + p_1 t) - q_0 S t e^{p_1 t}/L + \frac{1}{L}\int_0^t e^{p_1(t-u)}\{1 + p_1(t - u)\}e(u)du \tag{42}$$

as the general solution for the case of equal roots.

VI. THE GENERAL n MESH CIRCUIT.

Let us now consider the general n mesh circuit subject to n driving electromotive forces acting on the various meshes and impressed at $t = 0$. If we introduce a columnar current matrix defined by the relation:

$$\{q\} = \int_{-\infty}^t \{i\}dt. \tag{43}$$

The canonical equations of the circuit become:

$$[L]\frac{d}{dt}\{i\} + [R]\{i\} + [S]\int_{-\infty}^t \{i\}dt = \{e(t)\}. \tag{44}$$

Now by the integration of the various elements of the matrix $\{i(t)\}$ we may form a matrix $\{I(p)\}$ whose elements are the Laplacian transforms of the elements of the matrix $\{i(t)\}$. The matrix $\{I(p)\}$ is defined by the equation:

$$\{I(p)\} = p\int_0^\infty e^{-pt}\{i(t)\}dt, \tag{45}$$

and for brevity, we will write:

$$\{I(p)\} \doteqdot \{i(t)\}. \tag{46}$$

This is a matrix generalization of (9). Similarly we introduce a matrix $\{E(p)\}$ whose elements are the Laplacian transforms of the elements of the matrix $e(t)$ and we then have:

$$\{E(p)\} \doteqdot \{e(t)\}. \tag{47}$$

In view of the transforms

$$\frac{d}{dt}\{i\} \doteqdot p\{I\} - p\{i^0\} \tag{48}$$

a generalization of (14) and

$$\{q\} \doteqdot \{I\}/p + \{q^0\} \tag{49}$$

a generalization of (15) where $\{i^0\}$ is a columnar matrix whose elements are the initial currents and $\{q^0\}$ is a columnar matrix whose elements are the initial mesh charges, we transform (44) into the following form:

$$[f(p)]\{I\} = p^2[L]\{i^0\} - [S]\{q^0\}p + p\{E(p)\}, \tag{50}$$

where

$$[f(p)] = p[Z(p)] \tag{51}$$

and

$$[Z(p)] = p[L] + [R] + [S]/p. \tag{52}$$

This is a generalization of equations (16), (17) and (18). If we premultiply both sides of (50) by $[f(p)]^{-1}$ the inverse of $[f(p)]$, we obtain:

$$\{I\} = [f(p)]^{-1}(p^2[L]\{i^0\} - [S]\{q^0\}p + p\{E(p)\}). \tag{53}$$

To proceed in the solution, let us introduce $[F(p)]$, the adjoint matrix of $[f(p)]$ defined by the equation:

$$[f(p)]^{-1} = \frac{[F(p)]}{D(p)} \tag{54}$$

where:

$$D(p) = |f(p)|. \tag{55}$$

The element F_{mn} in $[F(p)]$ is the cofactor of f_{nm} in the determinant $D(p)$. The matrices $[f(p)]$ and $[F(p)]$ have

the property that:

$$[f(p)][F(p)] = [F(p)][f(p)] = [U]D(p), \tag{56}$$

where $[U]$ is the unit matrix of order n. We thus have:

$$\{I\} = \frac{[F(p)]}{D(p)}(p^2[L]\{i^0\} - [S]\{q^0\}p + p\{E\}). \tag{57}$$

Before proceeding to a computation of the inverse transform of $\{I(p)\}$, it is well to realize that there are several cases to be considered. In the first case we have the standard type of problem in which the roots of $D(p) = 0$ are all distinct and the degree of $[F(p)]$ is less than that of $D(p)$. A second case arises when the roots of $D(p) = 0$ are distinct but the degree of $[F(p)]$ is equal to the degree of $D(p)$. The third case is where the degree of $[F(p)]$ is greater than the degree of $D(p)$. The fourth case is the case of multiple roots of $D(p) = 0$. For completeness, we shall discuss the several cases in turn.

1. *Case I.*

In the first case we have the usual case arising in problems of the standard type. This is the case most frequently met in actual practice. The determinantal equation $D(p) = 0$ then has $2n$ distinct roots $p_1, p_2, \cdots, p_{2n}$ and the degree of $[F(p)]$ is less than that of $D(p)$. Since, in general, the degree of $D(p)$ is $2n$ and the degree of $[f(p)]$ is 2, it follows that the degree of $[F(p)]$ is $2(n-1)$. In very special cases, however, this may not be true.

In case the degree of $[F(p)]$ is less than that of $D(p)$ and the determinantal equation has $2n$ distinct roots, we may assume the following matrix identity:

$$[F(p)]/D(p) = \sum_{r=1}^{r=2n} [A_r]/(p - p_r), \tag{58}$$

where $p_1, \cdots, p_{2n}$ are the roots of $D(p) = 0$ and $[A_r]$ are square matrices of constants to be determined. To determine $[A_s]$ multiply both sides of (58) by $(p - p_s)$ and put $p = p_s$. We then have:

$$\lim_{p \to p_s} (p - p_s)[F(p)]/D(p) = [F(p_s)]/D'(p_s) = [A_s], \tag{59}$$

where

$$D'(p_s) = \frac{d}{dp} D(p) \Big\}_{p=p_s}. \tag{60}$$

We thus have the matrix identity:

$$[F(p)]/D(p) = \sum_{r=1}^{r=2n} \frac{[F(p_r)]}{D'(p_r)} \frac{1}{(p - p_r)}. \tag{61}$$

Now since:

$$p^2/(p - p_r) \doteq p_r e^{p_r t}, \tag{62}$$

we have:

$$\frac{[F(p)]}{D(p)} p^2[L]i^0 \doteq \sum_{r=1}^{r=2n} \frac{[F(p_r)]p_r[L]\{i^0\}e^{p_r t}}{D'(p_r)}; \tag{63}$$

this is a generalization of (22). Similarly, since

$$p/(p - p_r) \doteq e^{p_r t}, \tag{64}$$

we have:

$$\frac{[F(p)][S]q^0 p}{D(p)} \doteq \sum_{r=1}^{r=2n} \frac{[F(p_r)][S]q^0 e^{p_r t}}{D'(p_r)} \tag{65}$$

a generalization of (23). And by the result that:

$$p\{E(p)\}/(p - p_r) \doteq p_r \int_0^t e^{p_r(t-u)}\{e(u)\}du, \tag{66}$$

it follows that:

$$\frac{[F(p)]p\{E(p)\}}{D(p)} \doteq \sum_{r=1}^{r=2n} \frac{[F(p_r)]p_r}{D'(p_r)} \int_0^t e^{p_r(t-u)}\{e(u)\}du. \tag{67}$$

We thus have the general solution for the standard case:

$$\begin{aligned} i(t) = &\sum_{r=1}^{r=2n} \frac{[F(p_r)]p_r[L]\{i^0\}e^{p_r t}}{D'(p_r)} \\ &- \sum_{r=1}^{2n} \frac{[F(p_r)][S]\{q^0\}e^{p_r t}}{D'(p_r)} \\ &+ \sum_{r=1}^{r=2n} \frac{[F(p_r)]p_r}{D'(p_r)} \int_0^t e^{p_r(t-u)}\{e(u)\}du. \end{aligned} \tag{68}$$

This is the general solution for the problem of the standard type with arbitrary initial conditions and arbitrary electromotive forces impressed on the several meshes at $t = 0$. This may be considered as a generalization of (45). Practically all the problems that arise in practice are of this type and the above solution is particularly concise and convenient for computational purposes. The greatest part of the labor involved is in the solution of the determinantal equation $D(p) = 0$, this may be done by the method of reference (10). Another part of the labor is the evaluation of polynomials for complex values of their arguments. This will be discussed in section VII. of this paper.

Case II

In this case, the determinantal equation has distinct roots and the degree of $[F(p)]$ is equal to the degree of $D(p)$. Now instead of the matrix identity (58) we must assume an identity of the following form:

$$[F(p)]/D(p) = [K] + \sum_{r=1}^{r=2n} \frac{[F(p_r)]}{D'(p_r)(p - p_r)}, \tag{69}$$

where $[K]$ is a square matrix of constants of order n. To find $[K]$ we place $p = 0$ in the above identity. We thus obtain:

$$\frac{[F(0)]}{D(0)} = [K] - \sum_{r=1}^{r=2n} [F(p_r)]/p_r D'(p_r). \tag{70}$$

Therefore:

$$[K] = [F(0)]/D(0) + \sum_{r=1}^{r=2n} \frac{[F(p_r)]}{p_r D'(p_r)}. \tag{71}$$

Hence we may write the matrix identity in the form:

$$\frac{[F(p)]}{D(p)} = \frac{[F(0)]}{D(0)} + \sum_{r=1}^{r=2n} \frac{[F(p_r)]p}{p_r D'(p_r)(p - p_r)}. \tag{72}$$

In computing the inverse transforms for this case, we must

first consider:

$$(73)\qquad \frac{[F(p)]p^2[L]\{i^0\}}{D(p)} = \frac{[F(0)]p^2[L]\{i^0\}}{D(0)} + \frac{[F(p_r)]p^3[L]\{i^0\}}{p_rD'(p_r)(p-p_r)}.$$

Now the computation of the inverse transform of the first member on the right of (73) brings us to a consideration of the transform of p^n where n is a positive integer. Van der Pol (reference 8) has shown that:

$$(74)\qquad p \doteqdot \delta(t) \qquad p^2 \doteqdot \delta'(t) \qquad p^n \doteqdot \delta^{[n-1]}(t),$$

where the $\delta(t)$ functions are the Dirac functions and have the property that:

$$(75)\qquad \int_{-\infty}^{+\infty} \phi(t)\delta^{[n]}(t)dt = \phi^{[n]}(0) \text{ and } \delta^{[n]}(t) = 0 \text{ for } t > 0.$$

Accordingly we have the following transform:

$$(76)\qquad \frac{[F(0)]p^2[L]\{i^0\}}{D(0)} \doteqdot \frac{[F(0)]\delta'(t)[L]\{i^0\}}{D(0)}.$$

However, since the function $\delta'(t)$ has the property that:

$$(77)\qquad \delta'(t) = 0 \qquad t > 0,$$

we see that this term contributes nothing for t greater than zero. Now, since:

$$(78)\qquad p^3/(p-p_r) \doteqdot p_r^2e^{p_rt},$$

we obtain directly:

$$(79)\qquad \sum_{r=1}^{r=2n} \frac{[F(p_r)]p^3[L]i^0}{p_rD'(p_r)(p-p_r)} \doteqdot \sum_{r=1}^{r=2n} \frac{[F(p_r)]p_r[L]\{i^0\}e^{p_rt}}{D'(p_r)}.$$

Let us consider now the transform of the terms:

$$(80)\qquad \frac{[F(0)]p[S]\{q^0\}}{D(0)} + \frac{[F(p_r)]p^3[S]\{q^0\}}{p_rD'(p_r)(p-p_r)}.$$

Now

$$(81)\qquad \frac{[F(0)]p[S]\{q^0\}}{D(0)} \doteqdot \frac{[F(0)]\delta(t)[S]\{q^0\}}{D(0)}.$$

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But, since $\delta(t) = 0$ for $t > 0$, this term contributes nothing to the solution. Since,

$$p^2/(p - p_r) \doteqdot p_r e^{p_r t}, \tag{82}$$

we have

$$\sum_{r=1}^{r=2n} \frac{[F(p_r)]p^2[S]\{q^0\}}{p_r D'(p_r)(p - p_r)} \doteqdot \sum_{r=1}^{r=2n} \frac{[F(p_r)][S]\{q^0\}e^{p_r t}}{D'(p_r)}. \tag{83}$$

We must now compute the transform of the term:

$$\frac{[F(0)]p\{E(p)\}}{D(0)} + \frac{[F(p_r)]p^2\{E(p)\}}{p_r D'(p_r)(p - p_r)}. \tag{84}$$

Now we have the theorem that:

$$\text{if } \{E(p)\} \doteqdot \{e(t)\} \quad \text{then} \quad p\{E(p)\} \doteqdot \frac{d}{dt}\{e(t)\}. \tag{85}$$

Using this theorem, and the result that:

$$\frac{p^2\{E(p)\}}{(p - p_r)} \doteqdot p_r^2 \int_0^t p^{p_r(t-u)}\{e(u)\}du, \tag{86}$$

we then have:

$$\frac{[F(0)]}{D(0)}\frac{d}{dt}\{e(t)\} + \sum_{r=1}^{r=2n} \frac{[F(p_r)]p_r}{D'(p_r)} \int_0^t e^{p_r(t-u)}\{e(u)\}du \tag{87}$$

as the transform of the term (84). In view of the above results we may write the solution for the case in which the degree of $[F(p)]$ is equal to the degree of $D(p)$ in the form:

$$\begin{aligned}\{i(t)\} = {} & \frac{[F(0)]}{D(0)}\delta'(t)[L]\{i^0\} \\ & + \sum_{r=1}^{2n} \frac{[F(p_r)]p_r[L]\{i^0\}e^{p_r t}}{D'(p_r)} + \frac{[F(0)]}{D(0)}\delta(t)[S]\{q^0\} \\ & + \sum_{r=1}^{2n} \frac{[F(p_r)][S]\{q^0\}e^{p_r t}}{D'(p_r)} + \frac{[F(0)]}{D(0)}\frac{d}{dt}\{e(t)\} \\ & + \sum_{r=1}^{2n} \frac{[F(p_r)]p_r}{D'(p_r)} \int_0^t e^{p_r(t-u)}\{e(u)\}du,\end{aligned} \tag{88}$$

where $\delta'(t)$ and $\delta(t)$ are impulsive functions with the properties expressed in equation (75).

Case III.

In some rare cases, the degree of $[F(p)]$ is greater than the degree of $D(p)$. In this case, the correct matrix identity is:

$$\frac{[F(p)]}{D(p)} = [Q(p)] + \sum_{r=1}^{r=2n} \frac{[F(p_r)]}{D'(p_r)(p - p_r)}. \tag{89}$$

Here $[Q(p)]$ is a square matrix of order n whose elements are obtained from those of $[F(p)]$ by dividing them by $D(p)$. In general, the elements of $[Q(p)]$ will be polynomials in p. Now in computing the inverse transform of $I(p)$, the $[Q(p)]$ matrix will introduce dirac functions that have the properties of (75) in the terms for the initial charges and currents. These terms will vanish for values of t greater than zero.

The $[Q(p)]$ matrix will introduce derivatives of the electromotive forces. The terms arising from the second member of (89) are the same as those for the standard case given by (68). It is therefore only necessary to add the various necessary derivatives of the electromotive forces as required by the elements of $[Q(p)]$ to the solution of the standard case as given by (68).

Case IV.

This is the case where $D(p) = 0$ has one or more repeated roots. When repeated roots occur, a set of s roots equal to p_s will give rise to terms in the identity (61) of the form:

$$\frac{[B_1]}{(p - p_s)} + \frac{[B_2]}{(p - p_s)^2} + \cdots + \frac{[B_s]}{(p - p_s)^s} \tag{90}$$

for the expansion of $[F(p)]/D(p)$. Here $[B_j]$ are matrices of constants. If we write:

$$D_s(p) = D(p)/(p - p_s)^s, \tag{91}$$

it is easy to show that the coefficient B_n is given by the expression:

$$[B_n] = \frac{1}{(s - n)!} \left\{ \frac{\partial^{[s-n]}}{\partial p^{[s-n]}} \frac{[F(p)]}{D_s(p)} \right\}_{p=p_s}. \tag{92}$$

We have now to consider the transforms of terms of the type:

$$\frac{[B_n]p^2[L]\{i^0\}}{(p-p_s)^n}. \tag{93}$$

To do this, we use the expression:

$$p^2/(p-p_s)^n \doteq \frac{e^{p_s t}t^{(n-1)}}{(n-1)!}\{p_s + (n-1)t^{-1}\}. \tag{94}$$

The transform of:

$$\frac{[F(p)][S]\{q^0\}p}{D(p)} \tag{95}$$

involves terms of the type:

$$\frac{p}{(p-p_s)^n} \doteq \frac{e^{p_s t}t^{(n-1)}}{(n-1)!}. \tag{96}$$

The term involving the transform of the electromotive force yields terms of the type:

$$p\{E(p)\}/(p-p_s)^n \doteq \frac{1}{(n-1)!}\int_0^t e^{p_s u}u^{(n-1)}\{p_s + (n-1)u^{-1}\}e(t-u)du. \tag{97}$$

We have thus covered all possible cases which might arise. Usually, however, the simple standard case is the one met in practice.

VII. EVALUATION OF POLYNOMIALS FOR COMPLEX VALUES OF THEIR ARGUMENTS.

After the evaluation of the roots of $D(p) = 0$ has been made, a great deal of the remaining labor involved is the evaluation of polynomial expressions for the various values p_j of the roots of the determinantal equation. Since, in general, the values of the roots are complex, this means the evaluation of the various polynomial elements of $[F(p)]$ and the evaluation of $D'(p)$ for complex values of p. Direct substitution of the complex roots $p_1, \cdots, p_n$ into the polynomial expressions becomes very laborious when $D'(p)$ and the elements of $[F(p)]$ are polynomials of high degree. The work is considerably simplified if a procedure analogous to that of syn-

thetic division to evaluate polynomials of high degree for real values of their argument is employed. To be specific, let it be required to evaluate the polynomial:

$$F_{11}(p) = a_n p^n + a_{n-1}p^{n-1} + \cdots + a_1 p + a_0 \tag{98}$$

for $p = p_r = h + jk$.

We could, of course, divide $F_{11}(p)$ by $(p - p_r)$ synthetically and the remainder would give us $F_{11}(p_r)$. It is simpler, however, to divide through by:

$$(p - h - jk)(p - h + jk) = p^2 - 2hp + (h^2 + k^2). \tag{99}$$

If we let:

$$\beta = 2h \tag{100}$$

and

$$\alpha = -(h^2 + k^2). \tag{101}$$

The division may conveniently be carried out in the following tabular form:

a_n	a_{n-1}	a_{n-2}	a_{n-3}	$\cdots$	a_2	a_1	a_0
		αb_{n-2}	αb_{n-3}	$\cdots$	αb_2	αb_1	αb_0
	βb_{n-2}	βb_{n-3}	βb_{n-4}	$\cdots$	βb_1	βb_0	
b_{n-2}	b_{n-3}	b_{n-4}	b_{n-5}	$\cdots$	b_0	Q_0	R_0

In this table, the algebraic sum of three quantities occurs in the general term such as:

$$a_{n-2} + \alpha b_{n-2} + \beta b_{n-3} = b_{n-4}. \tag{102}$$

From this table, we obtain:

$$F_{11}(p_r) = p_r Q_0 + R_0. \tag{103}$$

The use of this process saves a great deal of labor. For a proof of the validity of this process reference (9) may be consulted.

VIII. CONCLUSION.

A form for the solution of the general n-mesh linear, lumped, bilateral network has been presented. The solution gives the currents in the various meshes when the system is

subject to initial mesh charges and mesh currents and is attacked by arbitrary electromotive forces at $t = 0$. By the use of matrix algebra it has been possible to discuss all the possible special cases that might arise in a concise manner. The foregoing solution is compact and formally complete. In practical applications to networks of many degrees of freedom it may well present formidable computational difficulties but by the systematization of the work by the use of matrix algebra the labor is diminished and no simpler general solution exists.

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Diamond Drilling Today.—H. J. LONGMORE. (*Mining and Metallurgy*, Vol. 21, No. 403.) More improvements have probably been made in the diamond-drill field in the past decade than were accomplished in the entire prior period since diamond drilling was discovered in 1864 by a French engineer named Leschot. True, machines were improved, and new accessories were adopted from time to time up to 1929, but no radical changes were made. The basic principle instituted during the early years of the industry were still in use in 1929. At that time, practically the only diamonds used were "black diamonds" or "carbon." These came from Brazil through a rather roundabout route, and by the time they got to the drilling trade their cost was excessive—$175 per carat and even higher at the peak. Perhaps because of this, or perhaps because of increased competition, the industry sought ways to reduce expenses even in the face of constantly increased labor costs due to higher wages and reduction in hours worked. The industry turned to the possibility of using "bortz diamonds," a crystalline form of diamond usually smaller and much cheaper than the carbon diamond, but still a diamond. At first, it was set by hand in the flat face type of bits previously used for the carbon. Rapidly it was discovered that, for some conditions, round face or "contour" bits worked much better. Then followed the development of "mechanically setting" the stones in the bits. This is also known as "cast setting" and today by far the majority of bits used in the diamond drill industry and by private industries employing their own drills, in the U. S., are the cast set bortz bits. The industry also considered improvement in diamond drill machines as a possible method of reducing costs. The result of this study has been astounding. Today we have faster, lighter, and more efficient machines than ever before, with increased capacities.

R. H. O.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

PECTIC SUBSTANCE OF COTTON FIBERS IN RELATION TO GROWTH.

It has long been known that cotton fibers represent single plant cells and that the growing fibers pass through certain distinct stages of development. The initial growth-phase consists in the development of the primary cell wall which serves as a protective coating for the cell during its subsequent development. With the completion of the primary wall, the secondary cell wall begins to develop. This process appears to consist in the deposition of cellulose. It is this secondary cell wall which is the commercially important part of the cotton fiber.

One part of the general research program, undertaken by the Textile Foundation at the Bureau on the chemical and physical properties of textile fibers, has been an investigation of the location and role of the various constituents of cotton fibers. The development of an analytical method for the accurate determination of uronic acids has made it possible to study in some detail the pectic substance of cotton. Since it has been suggested by other investigators that the pectic substance occurs mainly in the primary cell wall, it appeared that more exact information could be obtained by determining the percentage of petcic substance at various stages of growth during the development of the fiber. If it is assumed that the pectic substance is largely confined to the primary cell wall, it would be expected that very young cotton fibers would contain a high percentage of pectic substance and that the percentage would decrease as fiber development progressed, and would reach its lowest value at the time of fiber maturity.

The analysis of two series of cotton fibers by Roy L. Whistler and Albert R. Martin of the Textile Foundation,

* Communicated by the Director.

working with Carl M. Conrad of the Department of Agriculture revealed that during the period of fiber elongation and primary wall formation, which continues until the fiber reaches an age of 18 to 20 days, the percentage of pectic substance was at its highest value and remained approximately constant. At the end of this period the percentage of pectic substance suddenly began to decline rapidly. This point was taken to indicate the beginning of the second phase of fiber growth which consits in the rapid deposition of cellulose to form the secondary cell wall. The steady decline in the percentage of pectic substance, which continued until the fibers reached an age of about 35 days, indicated that cellulose was continually being deposited during this time. From the 35th day until the time the boll opened little change in the percentage of pectic substance occurred. It was, therefore, assumed that at 35 days the growth of fibers had been practically completed, and that but little change occurred thereafter. The three stages of development are in good agreement with those deduced by other investigators from different types of data.

The complete report of this work will appear in the *Journal of Research* for September (RP1326) and in *Textile Research* and *American Dyestuff Reporter*.

X-RAY STUDIES OF SILK FIBERS.

Studies of the diffraction of X-rays by natural silk fibers have been made at the Bureau in order to gain further information about the complex internal structure of these fibers. The diffraction phenomena reveal the presence of definite crystalline arrangements of atomic or molecular units in parts of the fiber structure.

Work was done on a *Bombyx mori* (cultivated) type of silk and on a *Tussah* (wild) silk, which were found to differ markedly in their crystalline organization. These differences were measured for the main body of the fiber, the fibroin component, and the results expressed in terms of crystal lattice constants. The outer layer of the fiber, the sericin component was found to exist on the fiber in a lower degree of crystallization, relative to that of the fibroin component, and with a more random orientation of the minute crystallites.

A comparison was also made of the diffraction of X-rays by silks of hard and soft nature, but the data obtained did not reveal any marked differences in diffractive properties.

ABSORPTION OF WATER BY PLASTICS.

A method of measuring the absorption of water by plastics has been prepared by Subcommittee V of Committee D–20 on Plastics of the American Society for Testing Materials, working in coöperation with Subcommittees II and III of Committee D–9 on Electrical Insulating Materials. This method has been adopted as a Tentative Standard and replaces the short-time water absorption tests provided in D48–39 for molded insulating materials, D229–39 for sheet and plate materials, D349–39 for laminated round rods, and D348–39 for laminated tubes.

Experimental data obtained in an exploratory investigation at the Bureau on the absorption of water by plastics in connection with the preparation of the above test method are summarized in a paper by G. M. Kline, A. R. Martin, and W. A. Crouse, which will appear in the Proceedings of the ASTM. These data pertain to the effects of various conditioning procedures, times of immersion, and prior history of the specimens on the values observed for water absorption by plastics. The plastics examined were: Phenol-formaldehyde molded, cast, and paper-base laminated materials; urea-formaldehyde molded and paper-base laminated materials; casein; cellulose nitrate; cellulose acetate; ethylcellulose; polyvinyl chloride-acetate; polyvinyl butyral; polystyrene in molded and cast forms; polymethyl methacrylate; and cold-molded plastics of the bituminous and phenolic types.

Data are presented on the rates of absorption and desorption of water by these plastics for periods up to two years. The variations in the dimensions of the specimens that are produced by these changes in moisture content are recorded.

CORROSION CHARACTERISTICS OF WROUGHT ALUMINUM ALLOYS.

Duralumin alloys are susceptible to two types of corrosion. Corrosive attack may follow the crystal boundaries of the

metal and thus seriously reduce its strength and resistance to shock. On the other hand, material of the same composition that has been given a different heat treatment may be susceptible only to the pitting type of attack, which is generally less damaging to the strength of the alloy than the intercrystalline type. The customary method of determining the type of attack that may develop in duralumin is to immerse the material for several hours in a solution that accelerates corrosion, and then to examine the sample with a metallographic microscope. This method is time-consuming and destroys the sample. There is a need for a rapid, nondestructive test to determine whether, for example, the metal in the wing covering of an airplane is susceptible to intercrystalline corrosion.

Because the electrochemical potential of duralumin in a salt solution is different for similar samples that have been given different heat treatments, the measurement of these potentials has been suggested as a means of predicting the type of corrosion that may occur. To determine the reliability of such predictions, Hugh L. Logan has measured the potentials of many samples of material, some known to be susceptible only to pitting, and others known to be susceptible to intercrystalline corrosion. In the *Journal of Research* for September (RP1328), it is shown that samples having potentials more negative than −0.657 volt, against a saturated calomel electrode, were generally susceptible to intercrystalline corrosion. Conversely, materials having less negative potentials were generally immune to intercrystalline corrosion and susceptible only to pitting. As the test method is delicate and requires a scientifically trained operator, it is primarily useful in the laboratory and not in the shop.

PREFABRICATED WOOD-FRAME CONSTRUCTIONS.

Tests have been completed in collaboration with the Forest Products Laboratory, on 39 specimens of prefabricated wood-frame wall, partition, and floor constructions, submitted by American Houses, Inc.

The wall and load-bearing partition specimens were subjected to compressive, transverse, concentrated, impact, and

racking loads; the nonload-bearing partition specimens to impact and concentrated loads; and the floor specimens to transverse, concentrated, and impact loads. The results are presented graphically and in tables in Building Materials and Structures Report BMS47. Copies are obtainable for 10 cents from the Superintendent of Documents, Government Printing Office, Washington, D. C.

Heating by Reversed Refrigeration.—A. J. LAWLESS. (*Heating, Piping and Air Conditioning*, Vol. 12, No. 8.) In order that the all-electric heating and air conditioning installation in the new building of the United Illuminating Co. serving New Haven, Conn., Bridgeport and surrounding communities might be comparable in operating cost with either coal or oil for fuel, the reversed refrigeration cycle for extracting heat from 55° F. well water is employed. In the reversed cycle, the cooled medium is discarded from the system and the heat removed from the medium becomes the useful work of the system. In the simple air conditioning refrigeration cycle, for every 200 B.t.u. removed from the cooled medium, approximately 47 B.t.u. of equivalent electrical energy is consumed; allowing for outside losses, approximately 240 B.t.u. must be removed from the condensers. This indicated that for every B.t.u. of electrical energy used, 5.1 B.t.u. is given up by the condensers. In reversed cycle heating systems this ratio of performance is decreased somewhat because of the increase in compression ratio in order to obtain condensing temperatures sufficiently high for heating purposes. The overall ratio of the system is also further decreased slightly, depending upon the additional electrical energy for the pumps and other equipment performing at a 1 to 1 ratio. Factors influencing the practicality of the reversed refrigeration cycle for building heating include the availability of a source of outside heat, cost of electrical energy, and whether or not summer air conditioning is desired. In some industries where refrigeration is required for processing in winter, it would be possible to use the heat given up by the condensers for heating purposes. There are two natural sources from which heat may be extracted in winter: outside air and water. In cold climates, water becomes the only practical source and in the case under consideration private wells are used. In the winter the well water is cooled by the refrigeration evaporators and discarded; the heat removed is given up by the condensers to a closed system of hot water heating. In summer the well water is pumped through the evaporators, delivered to the various cooling units in the building, and then returned to the condensers and discarded.

R. H. O.

THE FRANKLIN INSTITUTE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections of the Institute any technical writings of members who have had occasion to publish such material.

Literary contributions from author-members will be gratefully acknowledged, properly inscribed and noted in the Journal of the Institute.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 8½ × 11 inches is forty cents.

The library and reading room are open daily from nine o'clock A.M. until five o'clock P.M. Saturdays until 12 o'clock noon. 603 readers made use of the facilities during the twenty-seven days of August.

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NOTES FROM THE BARTOL RESEARCH FOUNDATION.

A NOTE ON THE PARALYSIS OF SUB-THRESHOLD COUNTERS.*

BY

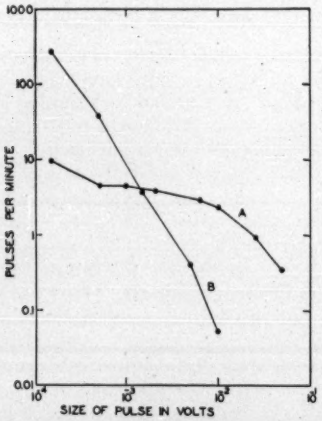
C. G. MONTGOMERY AND D. D. MONTGOMERY.

The paralysis of a Geiger counter apparatus upon exposure to high intensity radiation is a well-known phenomenon which is readily explained by the blocking action either of the vacuum tube circuits associated with the counter or, more commonly, of the counter wire circuit itself. When counters are operated below the potential above which they act as Geiger counters, another and more subtle variety of paralyzing action may take place. Counters used at potentials below the Geiger counter threshold, but with high enough potentials to have considerable ionization by collision, we designate as subthreshold counters. They are generally applied in a situation in which one wishes to count fairly large bursts of ionization in the counters, such as would be produced by alpha-particles, for example, against a background of smaller bursts produced by electrons from accompanying gamma-rays. If the background has a small intensity, it is possible to arrange that the voltage pulses from the counter wire caused by the alpha-particle ionization as increased by collision are, say, at least ten times as large as the largest pulse caused by the electrons, and we can achieve quite satisfactory counting conditions. In order to have the pulses from the alpha-particles large, the potential across the counter is maintained above the region where there is strictly proportional gas amplification, but not sufficiently high to destroy the discrimination of the counter. However, if the background of electrons is too intense, an alpha-particle will sometimes occur in the counter before the ionization resulting from the passage of electrons at some time previously has been completely swept out of the counter. If this be the case, the

* Reprinted from *The Review of Scientific Instruments*, 11, 237 (1940).

field near the wire is reduced, and consequently the amplification of the ionization produced by the alpha-particle is lessened, possibly even to the extent that the alpha-particle pulse is reduced in size below the minimum value necessary for it to be recorded. The pulse-size distribution curves shown in Fig. 1 illustrate to what extent such conditions may

FIG. 1.



Pulse-size distribution curves, on logarithmic scales, for the alpha-particles from the walls of a group of counters operated below the Geiger counter threshold. The ordinates give the number of counts per minute whose size, expressed in volts on an individual counter wire, is greater than the size given by the abscissae. Curve *A* was taken with no source of ionization present except the natural background, curve *B* was taken with two milligrams of radium 50 cm. from the counters.

prevail. A group of 25 counters, each with a cathode 20 cm. long and one cm. in diameter and filled with 18 cm. of a gas mixture composed of 94 per cent. argon and 6 per cent. oxygen, were connected in parallel and to a linear amplifier and recorder. A potential of 866 volts was applied to the counters, which potential is about 125 volts below the Geiger counter threshold. Curve *A* shows the number of pulses greater than a given size as a function of that size when the counters were exposed only to cosmic radiation and the natural radioactivity of the surroundings. The observed counts

were almost entirely caused by alpha-particles ejected from the radioactive contamination in the counter walls, although some of the smallest ones were caused by slow electrons. Curve *B* gives the distribution of pulse sizes when two milligrams of radium were placed 50 cm. from the counters. We see that while the number of small pulses was much increased, the number of large pulses was considerably decreased, as a result of the decrease in size of the pulses by the presence of the large amount of background ionization. From these data it is evident that care should be exercised in the use of a counter operating in this manner to insure that either the background radiation has no effect upon the size of the larger pulses which one desires to observe or that the background is kept constant during the measurements, particularly so when the counters are used near a cyclotron where the intensity of the gamma-radiation is extremely high. The actual magnitude of the suppression of the gas amplification of the large pulses depends considerably upon the experimental conditions. The suppression is reduced by using counters of small diameter, at low potentials, with large wire capacities. It is evident that the magnitude of the effect is almost independent of the circuit used to record the pulses, since the effect depends upon the reduction of the *field strength* near the counter wire, and not upon the wire *potential* or any of the potentials of the circuit components as is the case in the phenomenon of the paralysis of an ordinary Geiger counter.

SHOWER PRODUCTION BY MESOTRONS IN DIFFERENT MATERIALS.*

BY

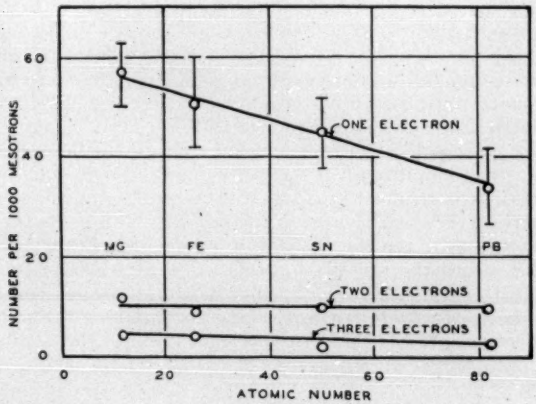
W. F. G. SWANN AND W. E. RAMSEY.

The apparatus used has already been described [2] and comprises a number of Geiger counter trays, each forming a sensitive area 20 by 20 cm., mounted in a vertical column and arranged in relation to the associated electrical circuits in such manner that the apparatus is only allowed to record when a ray passes through all of the trays. Below the top

* Reprinted from *The Physical Review*, **57**, 749 (1940).

tray is placed a block of lead 18 cm. thick, or its mass equivalent in other materials, so that all records by the apparatus are initiated by a ray which is capable of passing through at least this thickness of material. When a ray passes through all of the trays, any shower rays accompanying it are recorded separately by the individual counters which they excite, the record being a photograph of spots of light from the mirrors of a number of electroscopes—one for each counter. The

FIG. 1.



whole apparatus is mounted below a 30-foot column of water contained in a large water tank.

With the above apparatus the authors measured the numbers of one-ray showers, two-ray showers, three-ray showers, etc., produced in lead last August, and the *present experiments extend the measurements to lead, tin, iron, and magnesium.*

In dealing with the various materials, it was necessary to have a thickness sufficiently great to insure that shower production through mesotrons had attained equilibrium. Thus, it was necessary to have the thickness comparable with or greater than the range of the electrons. In order that one might be free from complications involved in variation of

mesotron intensity by different absorptions in the different elements, it was arranged that, in the case of each element, the total equivalent thickness as regards mass absorption should be the same, the balance being adjusted for convenience in each case by the choice of a suitable thickness of lead above the material under examination.

Figure 1 shows the results. Horizontally are plotted the atomic numbers of the elements concerned. The ordinates represent the numbers of showers, of the kind cited, associated with the passage of 1000 mesotrons through the apparatus.

The two-electron, and perhaps the three-electron, showers showed frequency of occurrence which is independent of the atomic number within the limits of accuracy of the experiments; but the one-electron showers show a marked dependence upon atomic number, a dependence sufficiently strong to dominate the situation even if we should plot a curve representing the total number of shower rays associated with 1000 mesotrons. These results are not in harmony with the elementary conclusions from Bhabha's theory, which predicts results practically independent of atomic number. As shown in our former paper,[1] however, the data for the actual numbers of showers agree, as regards order of magnitude, with the theoretical predictions.

It is a curious fact that the regularity of the graphs for the actual data is greater than one might expect on the basis of the statistical uncertainty indicated in the usual way by the vertical lines. This is a matter which may have a rather profound significance in relation to processes involved.

[1] Presented at the New York Meeting of the American Physical Society, February 23–24, 1940 (*Phys. Rev.*, **57**, 554(A) (1940)).

[2] W. F. G. Swann and W. E. Ramsey, *Phys. Rev.*, **56**, 378 (1939); W. F. G. Swann, *Rev. Mod. Phys.*, **11**, 242 (1939).

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

The Molecular Basis of Resin Behavior.—ELMER O. KRAEMER. Condensation of paper presented at Gibson Island Conference on Organic High Molecular Weight Compounds, June 8, 1940. (Continued from page 402.)

Indirect Methods for Determining the Molecular Weights of Macromolecules.—The various indirect methods for determining macromolecular weights all require calibration by means of one of the absolute methods already discussed, *using the specific materials to which the indirect method is to be applied.* By "specific materials" we mean macromolecules not only of the same chemical composition, but also prepared in the same manner. As will appear below, disregard of this fact could lead to errors of several hundred per cent. in the determination of relative molecular weights of polystyrenes by the viscosity method, and the same situation doubtless exists for other synthetic polymers.

1. The most important indirect method for macromolecular weights is Staudinger's viscosity method, which is applicable to the members of any polymer-homologous series of elongated or non-spherical molecules when the molecular shape is well-defined and "analogous" throughout the series. For linear and some branched molecules containing substantially only one kind of polymer-unit, Staudinger's viscosity rule may be expressed by the equation $P = K([\eta] - a)$ where P is the degree of polymerization, $[\eta]$ is the intrinsic viscosity, and K and a are constants specific for the particular polymer-homologous series. ($K = 1/10K_m$, where K_m is Staudinger's viscosity constant). According to recent work from Staudinger's laboratory, the relationship betewen P and $[\eta]$ is not quite linear for some branched chain molecules, as the polymeric vinyl derivatives. In any event, for large values of $[\eta]$, the effect of a becomes negligible, and, in fact, this constant is usually ignored. K not only varies with macromolecules of different chemical composition, but may

also vary with different methods of polymerization of a given material. Thus, according to Staudinger, K varies between 600 and 2000 for polystyrenes made at different temperatures, owing to different degrees of molecular branching.

There still is some uncertainty concerning the absolute values of K, as revealed by the following table:

	Our Values (by Ultracentrifuge)	Staudinger's Values (by Osm. Press, etc.)
Cellulose in cuprammonium	260	200*
Cellulose acetate in acetone	230	110
Nitrocellulose in acetone	270	90
Ethyl cellulose	300	100
Methyl cellulose	—	100
Rubber in ether	3000	300
Rubber in benzene	2000	—
Starch acetate in cresol	—	1000
Starch in formamide	—	1700
Arabo-galactan in water	—	4000

* About two years ago, the value given was one-half as large.

As will be discussed in greater detail below, all these macromolecular materials are non-uniform, and as a consequence, the kind of average molecular weight obtained depends upon the relationship between molecular size and the physical property measured. Briefly, we have shown that the viscosity method, when it is applicable, gives "weight-average" values (for definition of terms, see Kraemer in "The Ultracentrifuge" by Svedberg and Pedersen, Oxford Press, 1940), whereas osmotic pressures give "number-average" values. The former is always larger than the latter, and in many cases of synthetic polymers (see papers by Flory) as well as experimentally fractionated products, the weight-average value is approximately twice the number-average value. In such instances, the value of K, determined by osmotic pressure, would therefore be only one-half as large as the correct value.

At one time Staudinger thought that K/n is a constant, where n equals the number of chain atoms in the polymeric unit, but since it is now realized that strictly linear macromolecules are relatively rare, it is uncertain to what extent this relationship is true. In any event, since viscosity is fundamentally a volume phenomenon, there is no reason to

expect simple relations between the K's for different types of polymers whenever their specific volumes are not identical. If there were no solvation, the "volumetric intrinsic viscosity" (i.e., $[\eta]_V = [\eta]/V$, where V is the specific volume of the dissolved macromolecules) would place viscosities on a comparable basis, differences being then solely due to differences in molecular volume and shape. The relative values of K would then differ correspondingly from those given above. Thus, our values of K for cellulose acetate and nitrate, reduced to the volumetric basis, are equal within the limits of experimental error. If, as Staudinger thought, the viscosity should be proportional to the number of chain atoms, the ratio of the maximum molecular length (considered as a stiff rod) to $[\eta]_V$ should be constant, but as shown by the following table, this is not actually so.

Polyhydroxydecanoic acid	1200
Cellulose (in cuprammonium)	840
Cellulose acetate	790
Rubber (in ether)	14000
Rubber (in benzene)	9000
Polychloroprene (in *n*-BuCl)	8400
Polychloroprene (in benzene)	6400

The variation may be due to molecular coiling, branching molecules, or differences in solvation measured volumetrically.

2. A second and important indirect method for molecular weights depends upon Flory's discovery that the viscosity of molten linear high polymers is given by the expression $\log \eta = D + B/T + CM^{0.5}$ where D, B, and C are constants specific to the material in question, B being practically independent of molecular weight in a given polymer-series. The method is particularly important since it involves the properties of a pure phase rather than of a solution. It is so far apparently limited to linear polymers and has been applied only to polyesters, but it is to be hoped that Flory will extend the method to other materials. All thermoplastic high polymers should be examined from this point of view, even though they can not be completely melted, since the temperature of plastic flow is evidently dependent upon molecular weight. The molecular weight obtained for a non-uniform material by Flory's method is a weight-average value.

3. An interesting method for relative molecular weights has been worked out by Polson, who makes use of the theory of the viscosity of a suspension of ellipsoids to determine the frictional ratio of macromolecules. The frictional ratio, in turn, is the constant of proportionality by which the diffusion constant must be divided (in accordance with equations by Perrin) in order to obtain the diffusion constant of spheres of the same volume. Therefore, ignoring solvation, one can in principle combine viscosity and diffusion measurements to give the molecular volume and, if the specific volume is known, the molecular weight. Results for proteins did not quite accord with the theory, but the theoretical equations were adjusted by means of a single constant, which may represent the effects of solvation. The resulting empirical equation was found to be applicable only to near-spherical protein with equivalent axial ratios not exceeding about 10.

4. As Huggins has shown theoretically and Lowell and Hibbert have found experimentally, the melting point in an homologous series of linear polymers is a function of molecular weight, and consequently, after the relationship is once established for a given series, it may be used in estimating molecular weights in the same series. The method is unfortunately not generally useful, for the melting points of high polymers are commonly not well defined, and the rate of change of melting point with molecular weight for high polymers is small.

5. For a few proteins it has been found possible to estimate the molecular weights by the study of the dielectric dispersion of their solutions and application of the theory of rotatory diffusion, from which characteristic relaxation times can be determined. If only one such relaxation time can be found, one may assume that the molecules are uniform spheres, and their radius can be calculated. If two relaxation times are found, and they are ideal with respect to the frequency variation of polarization, one may infer the presence either of two spherical molecular species of different size, or of a single species of non-spherical molecules having two characteristic axes. By considering the molecule equivalent to an ellipsoid and assuming the specific volume of the equivalent ellipsoid to equal the partial specific volume of the macromolecule,

one may calculate a molecular volume and a molecular weight. If the molecule is not truly an ellipsoid, the molecular volume is apt to be in error, and in such a case, the specific volume assumption being also inaccurate, the molecular weight value would be doubly affected. In any event, the molecular volume calculated includes any bound solvent. If there is a continuous spread of relaxation times, it means that either the molecules or particles are non-uniform in size or the magnitude of the dipole moment, or that the particles are not rigid dipoles. With these uncertainties, it can be appreciated that the dielectric method is not an absolute one, and in general, the interpretation of dielectric dispersion results depends upon the availability of supplementary data. Preferably, the method should be considered as a confirmatory one.

6. The so-called end-group method, although frequently used in the past to estimate molecular weights, is strictly speaking neither an absolute nor an indirect method susceptible to calibration. In order to make use of "end-group" determinations in estimating molecular weights, it is necessary to know how many end-groups there are per molecule. With but one characteristic group per molecule the accuracy of analytical methods commonly restricts use of the method to relatively small molecules, say to weights less than 20,000. In general, however, the number of end-groups per molecule is not positively known, and the true value of end-group determinations is in the elucidation of the structure of macromolecules. In this category belongs the valuable work of the Haworth school on polysaccharides.

7. In principal, any property of a macromolecular material that varies with molecular weight may, after calibration by means of an absolute method, be used to determine molecular weights. For instance, Spurlin has shown how the fold-resistance of nitrocellulose depends upon the degree of polymerization, and undoubtedly, similar results would be displayed by many high polymers.

(*To be continued.*)

Inhibition of the Succinoxidase System by Extracts of Tumour and Normal Tissues.—KENNETH ALLAN CALDWELL ELLIOTT (*Biochemical Journal*, **34**: 1134, 1940). It had been

observed previously by Elliott et al. (*Biochem. J.* **29**: 1937, 1935; **32**: 1407, 1938) that the "succinoxidase" activity of certain tumor tissues was very low. It seemed possible that the low activity might be due to the presence in the tumors of an inhibitor of the system as well as to a deficiency in the enzymes and carriers responsible for succinate oxidation.

It has now been found that the aerobic oxidation of succinate with liver suspensions is strongly inhibited by suspensions or extracts of certain tumors and also of some normal tissues, especially pancreas and spleen. The inhibitor is nondialyzable and, to some extent at least, thermolabile. Commercial trypsin in low concentration is also inhibitory. It is possible that the tissue inhibitor is a proteolytic enzyme.

The succinic dehydrogenase (anaerobic reduction of methylene blue with succinate) and the cytochrome oxidase (oxidation of *p*-phenylene diamine in the presence of added cytochrome *c*) are both inhibited by tumor extracts, but not strongly enough to account for the main effect of the inhibitor on the complete succinoxidase system of liver. Cytochrome *c* appears not to be affected. It is concluded that the inhibitor acts particularly strongly on some component of the succinoxidase system of liver, possibly cytochrome *b*.

Kidney suspensions are affected similarly to liver suspensions. The succinoxidase activity of brain and muscle suspensions is also inhibited. But the succinoxidase activity of heart suspensions is scarcely at all affected and tumor extracts actually provoke an acceleration of succinate oxidation with heart suspension in the presence of added cytochrome *c*. The oxidation of *p*-phenylene diamine with heart is considerably inhibited by tumor extract in the absence of added cytochrome *c*, but not in its presence. This may indicate that cytochrome *b* is affected in heart but that the rate of succinate oxidation by heart is not limited by cytochrome *b*.

Since certain tissue suspensions contain a rapidly acting enzyme inhibitor, caution must be used in drawing conclusions from work on "surviving" whole tissue preparations as to the quantitative distribution of enzyme systems and their biological role.

A Fraction of Human Urine which Causes Fetus Resorption in Rats and Mice.—J. O. ELY (*American Journal of*

Cancer, **39**: 379, 1940). The intravenous injection of small amounts of the urine of advanced cancer patients into rats or mice on the 14–16th day of pregnancy was found to cause vaginal bleeding and stupor, followed by resorption of the fetus and eventual recovery of the animals. Similar effects were obtained with the urine of a healthy person.

A fraction containing the active principle has been prepared from a number of samples of urine of several persons by the following method: Concentration of the urine sample to about ¼ of the original volume by placing it in a cellophane dialyzing tube and suspending the tube in a warm current of air. To the concentrated urine 4 volumes of ethyl alcohol were added. After centrifugation and discarding the supernatant fluid, as much as possible of the alcohol-insoluble precipitate was dissolved in distilled water. Approximately 5 grams of Norite per 100 cc. were added. After 5–10 minutes the solution was filtered through a Seitz clarification filter pad followed by filtration through the Seitz sterilization filter pad *EK*. The filtrate was then thoroughly dialyzed in distilled water.

Using fractions prepared as described, fetus resorption was obtained in 61 of 65 rats used in testing. Resorption was obtained in all of 18 mice used in testing 8 of the samples.

One of the fractions which was effective when tested gave, when analyzed by our Microchemical Department, the following results: Dry weight 0.03 per cent., ash 12 per cent. of the dry weight. Calculated on the ash-free basis the solid material contained 8.7 per cent. N, 8.7 per cent. H and 43.2 per cent. C. Traces of P and S were present.

Another active sample gave a positive reaction to Millon's test but negative reactions to ninhydrin, biuret and Voisenet tests. Spectrographic analysis showed an absorption band with a maximum at 2500–2700 Å indicating either a mixture of substances or a substance of complex composition.

BOOK REVIEWS.

PROCEEDINGS OF THE SEVENTH SUMMER CONFERENCE ON SPECTROSCOPY AND ITS APPLICATIONS, held at the Massachusetts Institute of Technology, Cambridge, Mass., July 17–19, 1939. 154 pages illustrations, tables, 19 × 25 cms. New York, John Wiley & Sons, Inc., 1940. Price $2.75.

Last year, in these columns, mention was made of the Sixth Summer Conference on Spectroscopy at the Massachusetts Institute of Technology. These proceedings of the Seventh Conference are bound in similar fashion. There are twenty-nine papers contained in this booklet, in somewhat abbreviated form and which were presented before an audience of 250 persons in Cambridge, Massachusetts. First there is a group of papers on the general background and methods of spectrographic analysis of materials; then there are papers on specific types of analysis; these are followed by papers on apparatus which has recently been developed for use in spectrographic analysis; then come papers on light sources and their study and control; then a group of papers on absorption spectrophotometry and its application to chemistry and medicine; and finally papers on miscellaneous spectroscopic subjects. The authors of the papers are all men of mark in this field of endeavor and they provide valuable information to all those interested.

R. H. OPPERMANN.

AN INTRODUCTION TO ASTRONOMY, by Robert H. Baker. Second Edition, 315 pages, plates, illustrations, 16 × 24 cms. New York, D. Van Nostrand Company, Inc. Price $3.00.

This is the second edition of a book well known among a host of followers of the subject. The general purpose and plan is similar to the 1935 edition but many parts have been revised in the interest of clarity, and the subject matter has been brought up to date.

The greater part of the book is devoted to a description of the solar system—the earth and its movements, the sphere of the stars, the moon and its phases, the paths of the planets, planets and their satellites, and the sun. Next, study is made of the stars as remote suns, stars and nebulae, the galactic system, and the exterior systems. Illustrative of the progress made in astronomy since 1935, the book mentions the tenth and eleventh satellites of Jupiter, the growing distrust of astronomers in the interstellar origin of the majority of meteors, and the moving pictures of solar prominences. Also the conclusion that most of the star clouds of the Milky Way are openings in the dust clouds is largely a product of the last five years.

The book is notable for its simpleness of treatment, its plain and clear explanatory nature with the avoidance of mathematical treatment, its illustrations and photographs, and likeable manner of approach. It may be read and understood by any person of average intelligence, in fact it makes a useful text, a good instrument for review and valuable reference.

R. H. OPPERMANN.

LIGHT AND COLOUR IN THE OPEN AIR, by M. Minnaert. 362 pages, illustrations, plates, 15 × 23 cms. London, G. Bell & Sons Ltd., 1940. Price 15s.

To have true appreciation of the wonderful works of nature one must first observe. Although such observation is through the senses, and the comparative limitation of the power of our senses is somewhat narrow, the efficiency of observation varies quite widely with individuals. As an example, few of us have ever taken particular notice of the shadow in the sunlight of a butterfly or a bird, that it looks like a round spot. Further, optical illusions are still prevalent, because often we fail to observe closely. Did you ever observe a moon rainbow, clouds of the purest colors such as green, purple-red or blue, the comparative visibility in air blowing from the sea and from the land? These and many more phenomena become visible to everyone who knows how, when and what to observe. They give rise to interesting questions which in many instances may be answered by further observation.

This book is an unusual one in that it aims to reveal phenomena which are partly things of everyday life and partly things as yet unseen by most of us although they are here for us to see. The domain covered is that part of nature not usually included by the naturalist—those subjects bordering on astronomy, geography, biology and physics. Everything described is within the average person's powers of understanding and observation. It is crammed full of explanations on how to make an abundance of observations with merely the use of the senses and without the use of instruments. Simple as well as complicated observations are given and grouped according to their mutual relationship such as sunlight and shadows, reflection and refraction of light, the curvature of light rays in the atmosphere, colors, rainbows, haloes, and coronæ, luminous plants, animals and stones. Extensive descriptive matter is given revealing how and why of these many occurrences.

Whether you think you are a keen observer of nature or not, there are many surprises in store for you here. It is a good book to develop culture, the powers of the senses, and to enhance appreciation of life.

R. H. OPPERMANN.

WHY SMASH ATOMS? by Arthur K. Solomon. 174 pages, illustrations, plates, 16 × 24 cms. Cambridge, Harvard University Press, 1940. Price $2.50.

As scientific research continues to advance, the question of its practical value is more often heard. This is in spite of the fact that almost daily there appear new products for the comfort, convenience, and use of man. The bridge between what has been known as pure research and practical development is still long to the layman although to the scientist it grows much shorter. There is, therefore, a fertile field for explanation, for education in the layman's language, that he may understand better the necessity for extension of scientific knowledge.

This book aims to accomplish this purpose with respect to the disintegration of the atom. It gives a clear idea of the tremendous forces required to smash the atom and to make new ones. Various methods of generating this force are explained as well as its concentration and use. The apparatus is described, supplemented with appropriate photographs, and the historical development of knowledge is laid out emphasizing the growth of knowledge through experience. Throughout this book the fundamental desires of the alchemists of old are taken

as the keystone for explaining the accomplishments of the modern scientist. What they desired in the use of the "philosopher's stone" and endeavored to make their public believe has today become a factual reality with us. This is the story of the scientific dissection of the atom, its examination, and results of the present and promise of the future.

The author is well qualified. He has taken a major part in the work itself, and the translation from the scientific to the layman's language is well done. It has been no easy task. The complexity of the concept of the structure of the atom is such that it sets the man of physics in this field apart from the man in the street. With the aid of the glossary in the back of the book the man with average intelligence can readily understand. The book is not "sugar coated" science in the sense of a short cut to knowledge, but it is popular science in a serious vein, serious to the extent of accomplishing an easy reading general explanation.

R. H. OPPERMANN.

PRINCIPLES OF ALTERNATING-CURRENT MACHINERY, by Ralph R. Lawrence. 678 pages, illustrations, 16 × 24 cms. New York, McGraw-Hill Book Company, 1940. Price $5.50.

Principles of Alternating Current Machinery is well known to a host of electrical engineers as their college text and as a useful reference book thereafter. It furnished the basis for practical engineering and led to many years of successful application. The very mention of an up-to-date edition of the book will demand attention as a tried and proven instrument for instruction.

The third edition retains the same sequence as the first and second. The transformer, because of it being more easily understood, is held up as a transformer of power and the alternator and motor are likened to it—as transformers of power, although the book stays strictly to the electrical features and does not go into the mechanical power with which the latter deal. The thought thus conveyed goes far to helping the student in a broad sense. But while this is evident in the book, the author begins with synchronous alternators, that subject being the one which has proven the best, in his experience, as a starting point. It is noted that in the interest of modernity the use of the direct axis and quadrature axis synchronous reactances is included. Also the old method of treating parallel operation of alternators from the conception of current interchange and synchronizing current has been replaced by the more general and more satisfactory method based on power angle.

Next, static transformers are discussed, their types, purposes, construction and operation. This includes a short section on multiwinding transformers. Synchronous motors and synchronous converters are treated, followed by polyphase induction motors which include the principles of power factor control and speed control by inserting voltages in the rotor circuit. The chapters devoted to single phase induction motors cover the determination of the operating characteristics of the motor by the use of symmetrical phase components and the capacitor motor. The last subject treated is series and repulsion motors, the former including a discussion of shunted interpoles.

Every subject is opened with descriptive matter of the machine and the reasons underlying its construction. Then the theory is taken up, first in a simple manner, leading to the more complex. The coverage is only of the most important types of alternating current machines, sufficient to give a general understanding

of the principles of all types. The author has been free in selecting what types should be developed in detail employing mathematical and analytical treatment.

Illustrations occur where necessary and there is a comprehensive subject index in the back. The book can be recommended for use as a text where prerequisites of the elements of alternating currents and the calculus have been met.

R. H. OPPERMANN.

CONVERSION OF PETROLEUM, PRODUCTION OF MOTOR FUELS BY THERMAL AND CATALYTIC PROCESSES, by A. N. Sachanen. 413 pages, illustrations, tables, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1940. Price $6.00.

The layman need only reflect on his experience with motor fuel today compared with the fuel of say 10 or 15 years ago, to realize the tremendous improvement. There are many other evidences of improvement to the engineer and many more very apparent to those engaged in or closely allied to petroleum refining. The pace of development has been fast, so much so that even those connected with the industry find it difficult to keep abreast of the voluminous literature almost daily appearing, as well as the constantly widening scope of the subject. It is to the end of the consolidation of the literature and the presentation of the fundamental principles and practices of converting petroleum products into gasoline, in the modern aspect, that this book is written. The author is associated with the Socony-Vacuum Oil Co. in the research and development division.

The book opens with a discussion on thermal and catalytic reactions of hydrocarbons which contains a very interesting table summarizing the cracking reactions of all hydrocarbons which may be present in cracking stocks. To give an idea of the literature behind this subject there is a bibliography of 152 references. The next in line for discussion are the fundamental factors of cracking and of hydrogenation.

A description is given of the equipment necessary for the conventional cracking processes used for treating liquid charging stocks, other processes, such as catalytic cracking or thermal conversion of gases employing the same conventional equipment with certain specific apparatus of special design. Following this, cracked gasoline is taken up and there is described its composition, physical properties and determinations of unsaturates and of napthenes and paraffins. The color of gasoline, gum, sulphur, octane rating, etc., are other subjects taken up here. There is a chapter devoted to the treatment of cracked gasoline in which various processes are explained. And lastly, cracked products other than gasoline are given space. These include cracked gases, recycle stocks and their use for furnace and diesel oils, high boiling cracked distillates, cracked residues, asphalts and coke.

The book is restricted in scope to the most important literature works as they seemed to the author; the elimination of references of historical value; and the avoidance of patent literature except in a few cases. The coverage within the specified limits, the numerous tables and drawings and the subject and author index in the back represent an exhaustive work which is valuable to everyone interested in these and related activities.

R. H. OPPERMANN.

PRINTING INKS, THEIR CHEMISTRY AND TECHNOLOGY, by Carleton Ellis. 560 pages, illustrations, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1940. Price $7.00.

One need only page through a few periodicals today to realize the tremendous advance made in the art of color work in printing. Illustrations in many colors and shades are beautiful to behold and impossible only a few years ago. In fact a little serious thinking when examining modern periodicals will bring forth a multitude of questions which must have been answered before the finished product was made. For instance, there are various grades of paper and surface finishes, printing was done in different parts of the country under different conditions of atmosphere effecting drying rates, etc., the accuracy and detailed composition, the mechanics of the work, the numerous colors and blends, and many others.

The chemist dealing with printing inks, of necessity, has played a very important role in this rapidly developing field. In casting about for treatises on the subject the author was struck with the very limited number available. The reason for this book, he says, is to fill the needs of chemists who are groping for information on printing inks and are unable to find the subject discussed in any comprehensive treatise dealing with it in a modern way. It, therefore, takes the form of a monograph, giving innumerable references throughout.

At the outset, printing inks are likened to paints and varnishes as coating compositions. Those not intimately familiar with the subject will be surprised at the similarities and will be interested in the peculiar problems described on the requirements of inks and the methods of using them. There is given a brief history of printing inks which is instructive to anyone.

With this as a background, printing ink vehicles are discussed, followed by driers, modifiers and pigments. Inks for the three general methods of printing (typography, lithography, and intaglio) are described. Somewhat of a departure from the general theme but important from the standpoint of completeness is the portion of the book devoted to photochemical preparation of matrices. There is a rather short discussion of the more important reproduction processes involving the use of sensitized surfaces, after which there is described the preparation of intaglio, relief, copy relief, and lithographic plates. Transfer inks, emulsion inks, special types of printing inks, printing ink problems, and the testing of printing inks are separate topics taken up later.

The book contains many illustrations and tables, and there is a glossary, a subject and a name index in the back. The author, whose versatility and thoroughness is well known, has produced here a book which is the result of an exhaustive examination of the literature coupled with an uncommon knowledge of the subject and an experienced hand at presentation. It is a modern exposition.

R. H. OPPERMANN.

MECHANICS OF LIQUIDS, AN ELEMENTARY TEXT IN HYDRAULICS AND FLUID MECHANICS, by Ralph W. Powell. 271 pages, illustrations, tables, 16 × 24 cms. New York, The Macmillan Company, 1940. Price $3.50.

There are a number of texts already available in this field. The unusual method of presentation and progression of this book suggests that that is the justification for its publication. But there is more than that. The author in the preface states, "During the past few years, there has been a great deal of discussion

of Fluid Mechanics versus Hydraulics. Thinking of these as two competing subjects is a great mistake. . . . The question is very closely analogous to the corresponding situation in the mechanics of materials (solids). The teacher of Strength of Materials does not consider the Theory of Elasticity as a competing subject, but as the primary control with which he must constantly check. . . . The writer believes that with the time available at most institutions, and with the possible exception of a few specialized curricula, it is wisest to limit the first course to non-compressible fluids—to what is here called the Mechanics of Liquids."

Thus it is seen that this text has an individual pedagogical background. The line of progression begins with elements of the subjects of hydrostatics and fluid flow and then passes to orifices and weirs, pipe flow, flow in open channels, a more advanced treatment on pipe flow, effect of viscosity and the use of models.

The treatment is a vigorous, practical one. The author uses the practice of working out problems as illustrations, and these and the text are supplemented by many problems for exercise, to which answers are supplied. Always it moves from the simple to the more complex.

The question of the suitability of the construction of this book to the classroom is one for teachers to decide. There are evidences that it at least should be given its "day in court." The book has definite value to the man who wishes to gain an insight into the principles or the fundamentals of the subject with which he may already have a working knowledge of facts and rules of practice.

R. H. OPPERMANN.

NATURAL COLOR PROCESSES, by Carlton E. Dunn. Third Edition, 232 pages, plates, 14 × 21 cms. Boston, American Photographic Publishing Company, 1940. Price $2.00.

The fascination and satisfaction obtained from the practice of photography is manifest from the enthusiasm exhibited by its many followers. Parallel to other similar fields of activity certain of its main branches are not available to all, but only to those who have passed through elementary stages underlying the principles of the art generally, and who have inclinations toward them. One of these branches is color photography. It is a specialized branch. The purpose of this book is to explain the various practical methods available for making photographs in natural colors, with as little theory as possible. As such its appeal is to the practical worker who has the background of photographic experience as a prerequisite. Testifying to the acceptance of this book among color photographers, is the fact that two editions have appeared previous to this one, in 1936 and 1938. This is also evidence of the progress of the art, and it is an indication of improvements in method of presentation and coverage.

At the outset there is given a simple analysis of color with relation to photography. This is somewhat brief but sufficient. The rest of the book is divided into eleven different sections including the making of color-separation negatives, Autotype Trichrome Carbro which is a method of making natural color prints on paper by means of superimposed red, blue and yellow pigment images. Belcolor printing film, the Chromatone process, reliefs and imbibition for color prints and transparencies, Duxochrome and Colorstil color printing films, dye mordanting, screen color transparencies, Dufaycolor, multilayer color processes, and the Finlay process.

Descriptions and directions given are quite to the point. There is little space wasted in non-essentials. Professionals and advanced photographers should find here helpful descriptions and directions. The lack of an index in the back is offset by a rather complete table of contents in the front.

R. H. OPPERMANN.

MAGNETISM AND VERY LOW TEMPERATURES, by H. B. G. Casimir. 93 pages, illustrations, 13 × 22 cms. Cambridge, University Press, 1940. Price $1.40.

The contents of this little book are based on a series of four lectures given at Cambridge in the spring of 1938 by the author on invitation of the late Lord Rutherford. The subject is one in which progress has been rapid and in recognition of this fact the author wisely stresses methods rather than results, the latter being somewhat of a preliminary nature.

The work is divided into seven parts. After an introduction which briefly sketches what the rest of the work details out, there is laid out the classical theory and experimental methods. Then there is given a summarization of the theory of paramagnetism in so far as is required for understanding the properties of those substances which are most useful in demagnetization experiments. Here the problem of calculating the energy, entropy and susceptibility as a function of temperature and of external field is solved where Curie's law is valid. A discussion of experiments on adiabatic demagnetization precedes the investigation of the deviations from Curie's law and the behavior of the specific heat when the temperature approaches to zero. This is followed by a description of some of the results obtained with various substances. Under the title of "relaxation phenomena," the last part of the book, the paramagnetic salt is treated as consisting of two coupled systems: on the one hand there is the system of magnetic ions, with its Stark splittings and magnetic interactions ("spin system"), on the other hand the system of lattice vibrations. Different possibilities are discussed.

The coverage is restricted to salts with a magnetism due to spins. It is a fitting addition to and characteristic of the Cambridge Physical Tracts.

R. H. OPPERMANN.

TWELFTH REPORT OF THE COMMITTEE ON CATALYSIS, NATIONAL RESEARCH COUNCIL. 388 pages, illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1940. Price $5.00.

The Committee on Catalysis of the National Research Council herewith present the twelfth report which is a somewhat critical and proportioned impression of the state of progress of catalysis, with a resumé of some of the important work published since the appearance of the previous report. Because of the change of topics under this heading of catalysis, the committee is reorganized after the completion of each report so that members may be chosen on the basis of their participation in major fields.

This twelfth report consists of some twenty papers on as many topics, written by members of the committee and several specialists. The papers summarize the present position, in the author's own words and opinion, of the topics. In other words, they are a review of the most important recent literature which have made contributions to the present state and trend of knowledge of each subject. In a sense, each is an abstract or digest written from an individual point of

view. Literature references from which material was taken are listed at the end of each paper. Of the twenty topics covered in this report, there may be mentioned activated adsorption; contact catalysis in the hydrogenation of coal, coal tar, and oil; progress in the catalytic hydrogenation of fats and oils; the synthesis of ammonia; retardation of chemical reactions; halide catalysis; biochemical catalysis.

In addition to these papers, there appears in the latter part of the report reviews of fifteen new books on catalysis which are also the work of the committee and are given in some greater detail than the average journal review.

The report is the result of exhaustive literature search and selection, as well as expert knowledge of the topics. Everyone interested in catalysis should examine this report. It is a means of keeping in touch with late contributions to an increasingly complex subject.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Silver in Industry, by Lawrence Addicks. 636 pages, illustrations, tables, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1940. Price $10.00.

Applied X-Rays, by George L. Clark. Third Edition, 674 pages, illustrations, 16 × 24 cms. New York, McGraw-Hill Book Company, Inc., 1940. Price $6.00.

Chamber's Technical Dictionary, by C. F. Tweney and L. E. C. Hughes. 957 pages, 15 × 21 cms. New York, The Macmillan Company, 1940. Price $5.00.

U. S. Department of Commerce, Coast and Geodetic Survey. Special Publication No. 219, Triangulation in New Mexico (1927 Datum), by Frank L. Culley. 306 pages, illustrations, 15 × 23 cms. Washington, Superintendent of Documents, 1940. Price 40 cents.

American Institute of Mining and Metallurgical Engineers. Transactions, Volume 137, Institute of Metals Division, 1940. 487 pages, illustrations, 16 × 23 cms. New York, Institute. Price $5.00.

Handbook of Mathematical Tables and Formulas, by Richard Stevens Burington. 275 pages, illustrations, 14 × 21 cms. Sandusky, Handbook Publishers, Inc.

Life on Other Worlds, by H. Spencer Jones. 299 pages, plates, illustrations, 14 × 21 cms. New York, The Macmillan Company, 1940. Price $3.00.

The Story of Superfinish, by Arthur M. Swigert, Jr. 672 pages, illustrations, plates, 16 × 24 cms. Detroit, Lynn Publishing Company.

Strength of Materials, by S. Timoshenko. Part I, Elementary Theory and Problems. Second Edition, 359 pages, illustrations, 16 × 23 cms. New York, D. Van Nostrand Company, Inc. Price $3.50.

Bell Telephone System, Monographs: B-1232, High-Precision Frequency Comparisons, by Larned A. Meacham. 8 pages, illustrations. B-1233, Spectrochemical Analysis of Dilute Solutions, by A. E. Ruehle and E. K. Jaycox. 4 pages, illustrations. B-1234, Room Noise at Telephone Locations—II, by D. F. Seacord. 7 pages, illustrations. B-1235, The Investigation of Synthetic Linear Polymers by X-Rays, by C. S. Fuller. 25 pages, illustrations. B-1236, A New Quartz-Crystal Plate, by W. P. Mason. 10 pages, illustrations. 5 pamphlets, 15 × 23 cms. New York, Bell Laboratories, 1940.